International Conference on
Recent Advances in Nonlinear PDEs and their Applications
in Celebration of the 60th Anniversary of CUHK

(October 30, 2023 - November 3, 2023)

TITLE & ABSTRACT

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On spiral strategies for blocking fire

Professor Stefano BIANCHINI
Scuola Internazionale Superiore di Studi Avanzati

The fire blocking problem can be stated as follows: the fire is propagating in all directions with speed 1 starting from a nonempty open set $\Omega_0 \subset \mathbb{R}^2$, and a barrier $\Gamma(t)$ is constructed with speed $\sigma > 0$, i.e. $L(\Gamma(t)) \leq \sigma t$. The question is if there a strategy to build a barrier $\Gamma(t)$ which encloses the fire in finite time.

It is known that for $\sigma \leq 1$ this is impossible, while for $\sigma > 2$ there is an admissible strategy. An open conjecture is that for $\sigma \leq 2$ the fire cannot be blocked.

A somewhat simplified conjecture is that if the blocking barrier is spiral-like, then one can block the fire only if the speed of building the barrier is $\sigma > 2.61....$ The positive part of this conjecture is known.

In this talk I will present how we prove (partially) the negative part, i.e. if $\sigma \leq 2.3$ then every admissible spiral is exponentially diverging, and in particular the fire cannot be blocked.

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From the Monge transportation problem to Einstein's gravitation through Euler's Hydrodynamics

Professor Yann BRENIER
CNRS, Universit Paris-Saclay

The quadratic Monge optimal transportation problem can be revisited in Euler's language of Hydrodynamics as was explained by Jean-David Benamou and the speaker about 20 years ago. It turns out that Einstein's theory of gravitation, at least in vacuum, can be treated in a very similar way as a kind of quadratic matrix-valued optimal transportation problem.

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Two compressible worlds that are not yet unified

Professor Didier BRESCH
Universite Savoie Mont Blanc

During this talk, I will discuss about recent mathematical results related to compressible flows with low regularity. I will comment around the gaps between two mathematical ingredients related to viscous compressible Navier-Stokes equations which have allowed to distinguish two families of results.

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Singularities in fluid: Self-similar analysis, computer assisted proofs and neural networks

Professor Tristan BUCKMASTER
New York University

In this presentation, I will provide an overview of how techniques involving self-similar analysis, computer assisted proofs and neural networks can be employed to investigate singularity formation in the context of fluids.

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Liouville type theorems in the stationary Navier-Stokes and the related equations

Professor Dongho CHAE
Chung-Ang University

We consider the stationary Navier-Stokes equations in $\mathbb{R}^3$

$$-\Delta u + (u \cdot \nabla)u = -\nabla p, \qquad \nabla \cdot u = 0. \tag{1}$$

The standard boundary condition to impose at the spatial infinity is

$$u(x) \to 0 \quad \text{as} \quad |x| \to 0. \tag{2}$$

We also assume finiteness of the Dirichlet integral,

$$\int_{\mathbb{R}^3} |\nabla u|^2 dx < +\infty. \tag{3}$$

Obviously (u, p) with $u = 0$ and $p =$constant is a trivial solution to (1)-(3). A very challenging open question is if there is another nontrivial solution. This Liouville type problem is wide open, and has been actively studied recently in the community of mathematical fluid mechanics. The explicit statement of the problem is written in Galdi's book[1, Remark X. 9.4, pp. 729], where under the stronger assumption $u \in L^{\frac{9}{2}}(\mathbb{R}^3)$ he concludes $u = 0$. After that many authors deduce sufficient conditions stronger than (2) and/or (3) to obtain the Liouville type result. In this talk we review various previous results and present recent progresses in getting sufficient condition in terms of the potential functions of the velocity.

References

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On the Development of Shocks in Compressible Fluids

Professor Demetrios CHRISTODOULOU
ETH-Zurich

In this talk, I shall discuss the content of my recent monograph *The Shock Development Problem* (EMS Monographs in Mathematics, EMS Publishing House, 2019), which addresses the problem of the development of shocks in a compressible fluid past the point of their formation. This problem is formulated in the framework of the Eulerian equations of a compressible perfect fluid as completed by the laws of thermodynamics. These equations express the differential conservation laws of mass, momentum and energy and constitute a quasilinear hyperbolic 1st order system for the physical variables, that is the fluid velocity and the two positive quantities corresponding to a local thermodynamic equilibrium state. Smooth initial data for this system of equations leads to the formation of a surface in spacetime where the derivatives of the physical quantities with respect to the standard rectangular coordinates blow up.

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Regularity and Turbulence

Professor Peter CONSTANTIN
Princeton University

I will discuss connections between turbulence and regularity theory for 3D Navier-Stokes equations.

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Accelerated propagation in the KPP equation with nonlocal diffusion and free boundaries

Professor Yihong DU
University of New England

A new phenomenon in nonlocal diffusion models is that accelerated propagation may happen, that is, the propagation speed could be infinite, which never occurs in the corresponding local diffusion model with compactly supported initial data. In this talk we will look at such a phenomenon for the KPP equation with nonlocal diffusion and free boundaries. For several natural classes of kernel functions appearing in the nonlocal diffusion term, we will show how the exact rate of acceleration can be determined. The talk is based on joint work with Dr Wenjie Ni.

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Gauss curvature type flows: convergence, stability and applications

Professor Pengfei GUAN
McGill University

The classical Gauss curvature flow was introduced by Firey in 1974. K.S. Chou proved the contraction to a point in finite time in 1985. B. Andrews established that the "shapes of worn stones" are round in dimension two in 1999. The same holds in high dimensions, by the works of Guan-Ni (convergence to soliton, 2016) and Brendle-Choi-Daskalopoulos (uniqueness of soliton, 2018). We will discuss some recent works on variation of Gauss curvature type flows built on aforementioned works: the stability of the flow with lower order perturbation, the convergence of anisotropy flows and relation to the L^p-Minkowski type problems.

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Inviscid Limit Problem of radially symmetric stationary solutions for compressible Navier-Stokes equation

Professor Itsuko HASHIMOTO
Kanazawa University

We consider an inviscid limit problem of radially symmetric stationary solutions for an exterior problem in $\mathbb{R}^n (n \geq 2)$ to compressible Navier-Stokes equation, describing the motion of viscous barotropic gas without external forces, where boundary and far field data are prescribed.

For both inflow and outflow problems, the inviscid limit is considered in a suitably small neighborhood of the far field state.

For the outflow problem, we prove the uniform convergence of the Navier-Stokes flow toward the corresponding Euler flow in the inviscid limit.

On the other hand, for the inflow problem, we show that the Navier-Stokes flow uniformly converges toward a linear superposition of the corresponding boundary layer profile and the Euler flow in the inviscid limit.

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Potentially singular behavior of 3D incompressible Navier-Stokes equations

Professor Thomas Y. HOU
California Institute of Technology

Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data is one of the most challenging problems in nonlinear PDEs. In this talk, I will present some new numerical evidence that the 3D Navier-Stokes equations develop nearly self-similar singular scaling properties with maximum vorticity increased by a factor of 10^7. This potentially singular behavior is induced by a potential finite time singularity of the 3D Euler equations. Unlike the Hou-Luo blowup scenario, the potential singularity of the 3D Euler and Navier-Stokes equations occurs at the origin. We have applied several blowup criteria to study the potentially singular behavior of the Navier-Stokes equations. The Beale-Kato-Majda blow-up criterion, the blowup criteria based on the growth of enstrophy and negative pressure, the Ladyzhenskaya-Prodi-Serrin regularity criteria all seem to imply that the Navier-Stokes equations develop nearly singular behavior. Finally, we present some new numerical evidence that a class of generalized axisymmetric Navier-Stokes equations with time dependent fractional dimension and nonlinear rotation force seem to develop asymptotically self-similar blowup.

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On the stability of homogeneous equilibria in the Vlasov-Poisson system on $\mathbb{R}^3$

Professor Alexandru IONESCU
Princeton University

I will discuss the topic of linear and nonlinear stability of homogeneous equilibria among solutions of the Vlasov-Poisson system in the Euclidean space $\mathbb{R}^3$. The Euclidean problem we consider here differs significantly from the classical work on Landau damping in the periodic setting, in several ways. Most importantly, the linearized problem cannot satisfy a "Penrose condition". As a result, our system contains resonances (small divisors) and the electric field can decay at most polynomially.

This is joint work with Benoit Pausader, Xuecheng Wang, and Klaus Widmayer.

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Eckhaus instability of the compressible Taylor vortices

Professor Yoshiyuki KAGEI
Tokyo Institute of Technology

This talk is concerned with the bifurcation and stability of the compressible Taylor vortex. Consider the compressible Navier-Stokes equations in a domain between two concentric infinite cylinders. If the outer cylinder is at rest and the inner one rotates with sufficiently small angular velocity, a laminar flow, called the Couette flow, is stable. When the angular velocity of the inner cylinder increases, beyond a certain value of the angular velocity, the Couette flow becomes unstable and a vortex pattern, called the Taylor vortex, bifurcates and is observed stably. This phenomenon is mathematically formulated as a bifurcation and stability problem. In this talk, the compressible Taylor vortices are shown to bifurcate near the criticality for the incompressible problem when the Mach number is sufficiently small. The localized stability of the compressible Taylor vortices is considered under axisymmetric perturbations and it is shown that the Eckhaus instability of compressible Taylor vortices occurs as in the case of the incompressible ones.

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Generalized Taylor-Couette flow

Professor Hideo KOZONO
Waseda University and Tohoku University

We consider the stationary Navier-Stokes equations in the region of two concentric rotating cylinders. We first show that if the Reynolds number is sufficiently small and if the flow is axi-symmetric with its L^∞-norm small, then it is necessarily our generalized Taylor-Couette flow. Furthermore, if the associated pressure is bounded in the vertical direction, in particular, periodic in z-axis, then it coincides with the well-known canonical Taylor-Couette flow. Next, we give certain bounds of the Reynolds number and the L^∞-norm of the velocity vector under which the fluid motion is necessarily axi-symmetric. This implies that the control of the Reynolds number and the bound of the velocity yield the unique laminar flow. On the other hand, from a viewpoint of PDE, our result gives a new exact solution of the Navier-Stokes equations, and prove the Liouville-type theorem around the non-trivial solution. This is based on the joint work with Profs. Yutaka Terasawa(Nagoya Univ.) and Yuta Wakasugi(Hiroshima Univ.).

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Shock formation for the Einstein-Euler system

Professor Jonathan LUK
Stanford University

I will discuss ongoing work in which we prove stable shock formation for a class of solutions to the Einstein-Euler system. This fits in the larger context of multi-dimensional shock formation for hyperbolic systems with multiple speeds, where the main novelty here is that the shock-forming wave propagates with a slower speed. This is a joint work in progress with John Anderson (Stanford).

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Optimal rate of convergence to nondegenerate asymptotic profiles for fast diffusion

Professor Yasunori MAEKAWA
Kyoto University

In this talk we consider the (possibly sign-changing) solutions to the Cauchy-Dirichlet problem for the fast diffusion equation in a bounded domain. It is well known that every weak solution vanishes at a finite time, which is uniquely determined by the initial datum. We discuss the rate of convergence and its optimality to the asymptotic profile at the extinction time when the profile is nondegenerate. This talk is based on the joint work with Goro Akagi (Tohoku University).

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On mathematical analysis of hard phase fluid with free boundary in relativity

Professor Shuang MIAO
Wuhan University

The hard phase model is an idealized model for a relativistic fluid where the sound speed approaches the speed of light. In this talk I will first review our results on the well-posedness for the free boundary problem of this model, then I will present our recent work on linear (in)stability for a family of steady states to this problem. This talk is based on joint works with Sohrab Shahshahani, Sijue Wu and Zeming Hao.

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Global wellposedness of general evolution PDE on the Fourier half space

Professor Kenji NAKANISHI
Kyoto University

This is joint work with Baoxiang Wang (Jimei & Peking). We consider the Cauchy problem for very general PDEs, where the time derivative of solution is written in terms of Fourier multipliers in space and analytic nonlinearity, with no other structural condition, but for initial data supported on the half space in the Fourier transform.

We prove the global wellposendess with no size restriction in a function space of distributions in the Fourier transform.

Besides the support condition, the function space requires decay at the boundary in the sense of measure, and uniform integrability in the orthogonal directions, but nothing else. In particular, the initial data may be more rough than the tempered distributions, grow polynomially at infinity, and contain mixture of periodic functions.

When the Fourier support is more restricted to a conical region, the generality of equations is extremely wide, including those locally illposed in the standard settings, and those with infinite derivatives and singularities in the nonlinearity. The global wellposedness holds even when the solution blows up in the sense of tempered distribution, or when the initial data is outside the domain of nonlinearity.

As more classical examples, our results may be applied to the incompressible and compressible Navier-Stokes and Euler equations, as well as to various nonlinear hyperbolic, parabolic and dispersive equations.

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Two short stories on dynamics in infinite dimensions

Professor Robert Leo PEGO
Carnegie Mellon University

I will report on (i) the resolution of a long-standing open problem regarding long-time behavior in a nonlocal gradient-flow model of phase separation, and (ii) striking formulas for solitary waves in Calogero-Moser lattices.

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Minimal surfaces in higher codimension

Professor Richard SCHOEN
University of California, Irvine

Many theorems and tools for dealing with minimal submanifolds work only in codimension one for minimizing or stable hypersurfaces. These include curvature estimates and related Bernstein theorems. For the two dimensional Plateau problem there are special existence results for surfaces of fixed topological type, and one can think of the absolute minimizing current spanning a boundary curve as a limit when one allows surfaces of higher and higher genus as competitors. That this process stabilizes and produces a regular surface with finite genus follows from curvature estimates in the codimension one case. In this talk we will discuss appropriate Bernstein theorems and their applications to this stabilization question.

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The Structure of the Maximal Development for Shock-Forming $3D$ Compressible Euler Solutions

Professor Jared SPECK
Vanderbilt University

I will discuss several of my joint works with L. Abbrescia on the maximal development for shock-forming solutions to the $3D$ compressible Euler equations. Roughly, the maximal development is the largest possible classical solution determined by the initial data. In our works, we derive the complete structure of a localized portion of the maximal development for open sets of initial data without symmetry, irrotationality, isentropicity, or strict convexity assumptions, for every equation of state aside from an exceptional one. In particular, we provide a complete description of a connected component of the initial singularity (which is a $2D$ spacelike submanifold), we describe the full structure of a localized portion of the singular set emerging from the initial singularity, and we describe the emergence of a localized piece of a Cauchy horizon from the initial singularity. Collectively, these results are a crucial step towards proving the uniqueness of the entire maximal development, at least for some open sets of initial data. One should note that for other hyperbolic PDEs, non-uniqueness of maximal developments is known to occur for some initial data. Our work builds on Christodoulou's breakthrough monographs on irrotational and isentropic solutions and my prior works with J. Luk, which revealed an implicit portion of the singular set. The key new ingredients are rough foliations of spacetime adapted to the shape of the boundary of the maximal development and a geo-analytic framework that yields suitable estimates on the foliations. Time permitting, I will discuss some of the many open problems in the field.

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Brakke's Mean curvature flow

Professor Yoshihiro TONEGAWA
Tokyo Institute of Technology

I will present some up-date on the mean curvature flow in the setting of geometric measure theory called Brakke flow. In particular, I describe some time-global existence of Brakke flow, the partial regularity theorems, and some on-going research. They are joint-works with either Lami Kim or Salvatore Stuvard.

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On the null controllability of nematic liquid crystal flows in dimension two

Professor Changyou WANG
Purdue University

In this talk, I will discuss a recent result on the null controllability of the simplified Erickson-Leslie (EL) system modeling the hydrodynamic of motions of nematic liquid crystal materials, under the zero Neumann boundary condition for the director d and the Navier-slip boundary condition for the fluid velocity u, over a two dimensional bounded, smooth region, via interior controls of u and controls by external magnetic fields for d. The ideas involve a rigidity inequality on H^2-maps into spheres under the zero Neumann boundary condition, the exact controllability of trajectories of EL system and Coron's null controllability of the Navier-Stokes equation under Navier slip boundary condition.

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A class of functionals with duality

Professor Xujia WANG
The Australian National University

We introduce a class of functionals subject to a duality restriction. The functional is of the form $J(U,V) = \int_U f(x)dx + \int_V g(y)dy$, where f, g are given non-negative functions. This model includes the Minkowski problem in the sphere and Kantorovich's dual functional in optimal transport as special cases. The Euler equations of the functionals are of Monge-Ampere type. In this talk, I will report some new results on the functionals, which were obtained in collaboration with Qiang Guang and Qi-Rui Li.

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On bounded Morse index solutions of Allen-Cahn on Riemann surfaces

Professor Juncheng WEI
University of British Columbia & CUHK

A recent result of Chodosh and Mantoulidis (Pub. IHES 2023) showed that the stationary varifolds arising from (a special) Allen-Cahn must be (embedded) geodesic networks (with possible multiplicities). In this talk, I will first discuss the construction and the computation of Morse index of concentrating solutions of geodesics with intersections (proving a conjecture of Chodosh-Mantoulidis). For desingularization we use 4-ended solutions of Allen-Cahn. For geodesics with higher multiplicties we prove that necessary and sufficient conditions are existence of bouncing Jacobi fields. With suitable conditions on bouncing Jacobi fields we construct and compute the Morse index solutions on multiplicity two (and higher) geodesics. For desingularization we use Jacobi-Toda system. (Joint work with Frank Pacard and Yong Liu.)

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Regularity in chemotaxis-fluid interaction

Professor Michael Gerhard WINKLER
Universitt Paderborn

Evolution systems coupling chemotaxis models to the Stokes and Navier-Stokes equations from fluid mechanics are considered. The focus is on the discussion of analytical approaches toward describing how far such systems, despite their considerable complexity, may inherit essential regularity features from the correspondingly unperturbed taxis-only and fluid-only subsystems. The presentation addresses both the case in which chemotactic interaction is determined by signal production through individuals, and models which alternatively account for signal consumption.

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On the stability of multi-dimensional rarefaction waves

Professor Pin YU
Tsinghua University

In his pioneering work in 1860, Riemann proposed the Riemann problem and solved it for isentropic gas in terms of shocks and rarefaction waves. It eventually became the foundation of the theory of one-dimension conservation laws developed in the 20th century. We prove the non-nonlinear structural stability of the Riemann problem for multi-dimensional isentropic Euler equations in the regime of rarefaction waves. This is a joint work with Tian-Wen Luo.