

**Deutscher Wetterdienst** 

Wetter und Klima aus einer Hand

# Nonlinear Methods for Data Assimilation and Inversion

#### **Roland Potthast**

**KLIMA UND UMWELT** 

FORSCHUNG



WETTER

**Roland Potthast September 2020** 

**Co-Authors:** 

LEISTUNGEN

DER DWD

Anne Walter Andreas Rhodin Nora Schenk Peter-Jan van Leeuwen

and many people of DWD Data Assimilation (FE12)

## Contents





**Prediction** (NWP): **Earth System Modeling!** 

- 2. Discussion of Ensemble (+Particle) Methods
- 3. Global+LAM+LES Model: ICON and ICON-EPS and

the LEKTF+EnVAR/KENDA System

4. LAPF & LMCPF Particle Filters for Non-Gaussian

Distributions – Details and Results







- Framework Operational Numerical Weather Prediction (NWP)
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#### **Framework Numerical Weather Prediction**







## **DWD Model Configurations**





## ICON Global, EU and D2





AIREP MODE-S PILOT TEMP DRIBU SYNOP RADAR LHN RADAR RW

**OPERATIONS** 

#### Development RADAR 3D SYNOP T,RH RADAR OBJ SEVIRI IR SEVIRI VIS Lightning LPI MWR GB Remote Crowd Source RADAR DualP

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- Framework Operational Numerical Weather
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## **Data Assimilation Cycling**





## **Data Assimilation Methods**



Since 2010

0.2

Why variational Data Assimilation (3D/4D-VAR)?



- Why Hybrid Methods? (3D/4D-EnVAR)
- Why Particle Filters? (PF,GPF,ETPF,LAPF,LMCPF) Since 2020



5. State Propagation



5. State Propagation



## **Stochastic View** $\Leftrightarrow$ **Minimization**



University of





### EDA: Ensemble Kalman Filter (EnKF)



Time t

Values

- Kalman Filter needs B update => expensive!
- Estimate B based on an ensemble of forecasted states (stochastic estimator).

B will be **flow-dependent** and variable, depending on the **model dynamics** and on the **observations** 



## **Ensemble Methods & PF**





## **3D-VAR, EnKF and PF**





## **3D-VAR, EnKF and PF**





## **Curse of Dimensionality**



- 1) Curse of Dimensionality
- 2) Low Ensemble Number
- **3)** Dynamical System Errors

Practically you cannot sample from highdimensional spaces!



## LAPF Basic Idea





## LAPF/LMCPF Basic Idea





## LAPF Development Summary Deutscher Wetterdienst

- Wetter und Klima aus einer Hand
- 1. Localized Adaptive Particle Filter (LAPF) first high-dimensional sampling tests in Summer 2015 (c.f. Inverse Modeling by Nakamura & Potthast)
- 2. Implemented in **DACE** (Data Assimilation Coding Environment) for the *global ICON* Model, first stable runs in 2016 by Anne Walter (c.f. Potthast, Walter, Rhodin MWR 2019)
- Extension to Localized Adaptive Mixture Coefficient Particle Filter (LMCPF) 2018 by 3. Walter and RP
- Implemented LMCPF for *global ICON Model* since 2018 (Walter, RP), Diskussions 4. with Peter-Jan van Leeuwen, Alternative PF for ICON+DACE
- 5. LAPF and LMCPF implemented in DACE for COSMO Convective Scale Model, first stable runs in Nov 2018 by RP
- Implemented LMCPF for ICON D2 (convective scale), stable runs in Nov 2019, by RP 6.
- 7. Lorenz 63 and 96 Implementations of LAPF/LMCPF and Testing (Nora Schenk & RP 2019 & 2020)
- 8. Implementation of **LMCPF** in Speedy Model by Miyoshi and Kotsuki, RIKEN (2019)
- 9. Testing LMCPF with SEVIRI VIS Reflectance Assimilation (2019/2020), Lilo Bach & RP

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- 1. Framework **Operational Numerical Weather** 
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### Global NWP Modelling: Det + EPS – Reality + Goals





## **Full Observation System**





## **Conventional Synop +**

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### **Observations: Geostationary Satellites**

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### **Observations: Polar Orbiting Satellites**





## <sup>28</sup> Ensemble Datenassimilation EnVar

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**Operational since January 2016** 















**Deutscher Wetterdienst** 



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Distributions – Details and Results



$$w_{\ell} := e^{-\frac{1}{2}(y - Hx^{(\ell)})^T R^{-1}(y - Hx^{(\ell)})}, \quad \ell = 1, \dots, L$$



# We need a selection based on relative weights!







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## Kalman Filter

**Best Estimator** 

$$x^{(a)} = x^{(b)} + BH^T (R + HBH^T)^{-1} (y - Hx^{(b)})$$

$$K = BH^T (R + HBH^T)^{-1}$$

$$\tilde{B} = (I - KH)B$$

**Kalman Matrix** 

**Update B Matrix** 

## **Ensemble B Estimator**

$$\bar{x} := \frac{1}{L} \sum_{\ell=1}^{L} x^{(\ell)}$$

$$B = \frac{1}{L-1}XX^T$$

$$X = (x^{(1)} - \bar{x}, ..., x^{(L)} - \bar{x}) \in \mathbb{R}^{n \times L}$$

Y := HX

 $Y^{T}(R + \gamma YY^{T})^{-1} = (I + \gamma Y^{T}R^{-1}Y)^{-1}Y^{T}R^{-1}$ 



## **Transformed Kalman Filter**

$$x^{(a)} = x^{(b)} + BH^{T}(R + HBH^{T})^{-1}(y - Hx^{(b)})$$
  
=  $x^{(b)} + \gamma XX^{T}H^{T}(R + \gamma HXX^{T}H^{T})^{-1}(y - Hx^{(b)})$   
=  $x^{(b)} + \gamma XY^{T}(R + \gamma YY^{T})^{-1}(y - Hx^{(b)})$   
=  $x^{(b)} + \gamma X(I + \gamma YR^{-1}Y^{T})^{-1}Y^{T}R^{-1}(y - Hx^{(b)})$ 

$$A = Y R^{-1} Y^T$$

$$B = \frac{1}{L-1} X X^T$$

## LAPF & LMCPF: Transform

 $\tilde{B}$ 



$$= (I - KH)B$$

$$= (I - BH^{T}(R + HBH^{T})^{-1}H)B$$

$$= (I - \gamma X X^{T}H^{T}(R + \gamma HX X^{T}H^{T})^{-1}H)\gamma X X^{T}$$

$$= X (I - \gamma Y^{T}(R + \gamma YY^{T})^{-1}Y)\gamma X^{T}$$

$$= X (I - \gamma (I + \gamma Y^{T}R^{-1}Y)^{-1}Y^{T}R^{-1}Y)\gamma X^{T}$$

$$= X ((I + \gamma Y^{T}R^{-1}Y)^{-1}(I + \gamma Y^{T}R^{-1}Y - \gamma Y^{T}R^{-1}Y)\gamma X^{T}$$

$$= X (I + \gamma Y^{T}R^{-1}Y)^{-1}\gamma X^{T}$$

$$= X (\frac{1}{\gamma}I + Y^{T}R^{-1}Y)^{-1}X^{T}$$
Transform for any  
Gaussian Particle

$$Y^{T}(R + \gamma YY^{T})^{-1} = (I + \gamma Y^{T}R^{-1}Y)^{-1}Y^{T}R^{-1}$$

## LAPF & LMCPF: Mixture





**Explicit Calculations possible for each term**  We need a selection based on relative weights!

## **LAPF & LMCPF: Weights**

**Projection onto Ensemble Space** 

**Derivation by** Anne Walter, Andreas Rhodin and RP (MWR 2019)

Abbreviating  $A := \mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y}$  and  $C := A^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b)$ 

**Projection Operator** 

$$P(\mathbf{y}^o - \overline{\mathbf{y}}^b) = \mathbf{Y}(\mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y})^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b),$$

**Projected discrepancy** 

$$P(\mathbf{y}^o - H\mathbf{x}^{(\ell)}) = \mathbf{Y}A^{-1}\mathbf{Y}^T\mathbf{R}^{-1}((\mathbf{y}^o - \overline{\mathbf{y}}^b) - \mathbf{Y}e_\ell)$$

Exponent = 
$$\mathbf{Y}(C-e_{\ell}), \ \ell=1,...,L.$$

$$P(\mathbf{y}^{o} - H\mathbf{x}^{(\ell)})]^{T}\mathbf{R}^{-1}P(\mathbf{y}^{o} - H\mathbf{x}^{(\ell)}) = [C - e_{\ell}]^{T}A[C - e_{\ell}], \ \ell = 1, ..., L,$$

Weights

$$w_{k,\ell} = ce^{-\frac{1}{2}[C-e_{\ell}]^T A[C-e_{\ell}]}, \ \ell = 1,...,L.$$

## **LAPF & LMCPF: Ingredients**



**Derivation by** Anne Walter, Andreas Rhodin and RP (MWR 2019)

**Ensemble Transform** 

Ensemble Transform as for LETKF X w<sub>m</sub>

Localization

Move Particle (Shift)

Localization on R as for LETKF  $A := \mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y}$  and  $C := A^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b)$ 

Move each particle as LETKF moves the mean, but now individually calculated

Adaptivity via O-B

Resampling

 $\mathbf{E}\left[\boldsymbol{d}_{o-b}^{T}\boldsymbol{d}_{o-b}\right] = \mathrm{Tr}(\mathbf{R}) + \rho \,\mathrm{Tr}(\boldsymbol{H}\boldsymbol{P}^{b}\boldsymbol{H}^{T})$ 

Modulated resampling in ensemble space around each remaining particle adaptively based on  $\rho$ 

## 4) LMCPF for Lorenz Examples





#### background • Lorenz 63 \* xbm **System** \* У Simulation analysis ٠ 26.4 **Prior Ensemble** $\ast$ truth **40 Members** LETKF 26.3 **Natural Run Sigma = 10** Truth Assimilation 26.2 **Posterior** Run Sigma = 10.5 **Ensemble Observation** 26.1 2.5 2.6 2.7 2.8 26 -1.2 -1.1 -1 -0.9 -0.8

## **Dynamical System Errors**

## **LMCPF for Lorenz Examples**





## **LMCPF for Lorenz Examples**







## Large-Scale Experimental Set-up

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- Full ensemble: 40 members
- Reduced resolution:
  - 26km deterministic
  - 52km ensembles
- Period: 01.05.2016 – 31.05.2016

Experiments programmed and carried out by Anne Walter, DWD& Uni Reading, and Roland Potthast, DWD& Uni Reading

In Cooperation with Peter-Jan van Leeuwen, Uni Reading

Global **RMSE** for **obs-fg** statistics (Radiosondes vs. Model) Period: 08.05.2016 – 31.05.2016



Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni Reading

## **LMCPF Scores vs LETKF**





Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni

Global **RMSE** for **obs-fg** statistics (Radiosondes vs. Model) Period: 08.05.2016 – 22.05.2016



Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni

## **LMCPF Scores vs LAPF**





Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni

## **New LMCPF Scores vs**





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1 5

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#### **LMCPF: Uncertainty based Move works**



# LAPF Spread vs LMCPF & LEets Ker Wetter dind Klima aus einer Hand



Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni

#### Statistics for spread at level 64 for variable T

Mean of spread



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Maximum of spread



time step

## **LMCPF Scores vs LETKF**

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2016/05/02 - 2016/05/24 INI: ALL UTC, DOM: ALL



Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni

## **LMCPF Scores vs LETKF**

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![](_page_53_Picture_3.jpeg)

![](_page_53_Figure_4.jpeg)

## LMCPF vs LETKF for ICON D2

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![](_page_54_Picture_2.jpeg)

![](_page_54_Figure_3.jpeg)

OPERATIONS AIREP MODE-S PILOT TEMP DRIBU SYNOP RADAR LHN RADAR RW

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![](_page_55_Picture_3.jpeg)

![](_page_55_Figure_4.jpeg)

Reflectance; Date = 20190603, 1400 UTC

![](_page_56_Figure_2.jpeg)

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6

**Deutscher Wetterdienst** 

Wetter und Klima aus einer Hand

**Deutscher Wetterdienst** Wetter und Klima aus einer Hand

![](_page_57_Picture_2.jpeg)

Reflectance; Date = 20190603, 1400 UTC

#### LMCPF Analysis Ensemble Mean

![](_page_57_Figure_5.jpeg)

**Observations** 

Evaluation by Liselotte Bach and RP Reflectance [analysis ensemble mean in observation space]; Date = 20190603, 1400 UTC

![](_page_57_Figure_9.jpeg)

![](_page_58_Figure_1.jpeg)

![](_page_58_Figure_2.jpeg)

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![](_page_58_Figure_3.jpeg)

**Deutscher Wetterdienst** Wetter und Klima aus einer Hand

![](_page_59_Picture_2.jpeg)

![](_page_59_Figure_3.jpeg)

#### **Graphics by**

Liselotte Bach Nora Amelie Schenk Christian Welzbacher

> Linear Analysis Distributions

VS

Non-linear Analysis Distributions

![](_page_59_Figure_9.jpeg)

## **Summary**

![](_page_60_Picture_2.jpeg)

# (1) LAPF and LMCPF extend the capabilities of the LETKF

- (2) Ensemble Transform, Localization, Adaptive Spread Control – LETKF, LAPF and LMCPF use the same tools
- (3) Gaussian Particles allow to move towards Obs in Ensemble Space
- (4) More flexible than LETKF
- Fully Non-Linear Filtering by LAPF and LMCPF
- (6) Stable Particle Filters for global and regional NWP
- (7) Simple to Code following LETKF
- (8) Promising Features for Cloud Assimilation

#### IOP Expanding Physics

#### **Inverse Modeling**

An introduction to the theory and methods of inverse problems and data assimilation

Gen Nakamura Roland Potthast

![](_page_60_Picture_15.jpeg)

## **Thank You!**

![](_page_61_Picture_2.jpeg)

![](_page_61_Figure_3.jpeg)