Liouville type theorems in the stationary Navier-Stokes and related equations

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Keywords: Stationary navier-Stokes equations, Liouville type theorem

We consider the stationary Navier-Stokes equations in $\mathbb{R}^3$

$$-\Delta u + (u \cdot \nabla)u = -\nabla p, \tag{1}$$
$$\nabla \cdot u = 0. \tag{2}$$

The standard boundary condition to impose at the spatial infinity is

$$u(x) \to 0 \quad \text{as} \quad |x| \to 0. \tag{3}$$

We also assume the finiteness of the Dirichlet integral,

$$\int_{\mathbb{R}^3} |\nabla u|^2 dx < +\infty. \tag{4}$$

Obviously (u, p) with $u = 0$ and p =constant is a trivial solution to (1)-(4). A very challenging open question is if there is another nontrivial solution. This Liouville type problem is wide open, and has been actively studiedrecently in the community of mathematical fluid mechanics. The explicit statement of the problem is written in Galdi's book[1, Remark X. 9.4, pp. 729], where under the stronger assumption $u \in L^{\frac{9}{2}}(\mathbb{R}^3)$ he concludes $u = 0$. After that many authors deduce sufficient conditions stronger than (3) and/or (4) to obtain the Liouville type result. In this talk we review various previous results and present recent progresses in getting sufficient condition in terms of the potential functions of the velocity.

References

[1] G. P. Galdi,: *An introduction to the mathematical theory of the Navier- Stokes equations: Steady-State Problems*, Springer, 2011.