EXISTENCE OF POSITIVE SOLUTION FOR A CRITICAL NEUMANN PROBLEM IN THE HALF-SPACE

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Abstract. In this talk, we concern with the existence of positive solutions for the following Neumann problem with critical growth

$$\begin{cases} -\Delta u - \dfrac{1}{2}(x\cdot\nabla u) = a|u|^{2^*-2}u + \mu|u|^{p-2}u & \text{in} \quad \mathbb{R}^N_+, \\ \dfrac{\partial u}{\partial n} = |u|^{2_*-2}u & \text{on } \partial\mathbb{R}^N_+, \end{cases} \tag{0.1}$$

where $\mathbb{R}^N_+ := \{(x', x_N) : x' \in \mathbb{R}^{N-1}, x_N > 0\}$ is the upper half-space, $N \geq 3$, $\mu \in \mathbb{R}^1$, $a \in \{0, 1\}$, $2 \leq p < 2^*$, n is the outward normal vector at the boundary $\partial\mathbb{R}^N_+$, $2^* = \frac{2N}{N-2}$ is the usual critical exponent for the Sobolev embedding $D^{1,2}(\mathbb{R}^N_+) \hookrightarrow L^{2^*}(\mathbb{R}^N_+)$ and $2_* = \frac{2(N-1)}{N-2}$ is the critical exponent for the Sobolev trace embedding $D^{1,2}(\mathbb{R}^N_+) \hookrightarrow L^{2_*}(\partial\mathbb{R}^N_+)$. By the Mountain Pass Theorem without (PS) condition and the delicate estimates for Mountain Pass level, we obtain the existence of a positive solution for different value of the parameters p and μ. Moreover, the nonexistence of positive solutions for problem (0.1) is also obtained by an improved Pohozaev identity and Hardy inequality according to the value of the parameters μ and p.

Date: January 18, 2024.