IMS PDE Workshop (Part I)

(January 26, 2024)

TITLE & ABSTRACT

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Existence of positive solution for a Critical Neumann problem in the half-space

Professor Yinbin DENG
Central China Normal University

In this talk, we concern with the existence of positive solutions for the following Neumann problem with critical growth

$$\begin{cases} -\Delta u - \dfrac{1}{2}(x \cdot \nabla u) = a|u|^{2^*-2}u + \mu|u|^{p-2}u & \text{in} \quad \mathbb{R}^N_+, \\ \dfrac{\partial u}{\partial n} = |u|^{2_*-2}u & \text{on } \partial\mathbb{R}^N_+, \end{cases} \tag{1}$$

where $\mathbb{R}^N_+ := \{(x', x_N) : x' \in \mathbb{R}^{N-1}, x_N > 0\}$ is the upper half-space, $N \geq 3$, $\mu \in \mathbb{R}^1$, $a \in \{0, 1\}$, $2 \leq p < 2^*$, n is the outward normal vector at the boundary $\partial\mathbb{R}^N_+$, $2^* = \frac{2N}{N-2}$ is the usual critical exponent for the Sobolev embedding $D^{1,2}(\mathbb{R}^N_+) \hookrightarrow L^{2^*}(\mathbb{R}^N_+)$ and $2_* = \frac{2(N-1)}{N-2}$ is the critical exponent for the Sobolev trace embedding $D^{1,2}(\mathbb{R}^N_+) \hookrightarrow L^{2_*}(\partial\mathbb{R}^N_+)$. By the Mountain Pass Theorem without (PS) condition and the delicate estimates for Mountain Pass level, we obtain the existence of a positive solution for different value of the parameters p and μ. Moreover, the nonexistence of positive solutions for problem (1) is also obtained by an improved Pohozaev identity and Hardy inequality according to the value of the parameters μ and p.

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Dynamics for liquid droplets and evaporation in inkjet printing of display thin films: Mathematical Modeling, problems and prospects

Professor Shijin DING
South China Normal University

This talk is concerned with the mathematical modeling and analysis for the key procedure in inkjet printing of advanced display thin films. Here the key procedure consists of three steps which are the jetting of polymer (OLED, for example) liquid droplets from the spray nozzle under piezoelectricity, motion and coalescence of droplets on the substrate, and the evaporation of the liquid thin film in pixel bars. The corresponding mathematical problems related to these three steps are free boundary problems for droplet-jetting from a nozzle and interface problems with solid-liquid-air moving contact lines and dynamical contact angles even with evaporating conditions in sessile domains for multi-component and multi-phase polymer fluids. We review and outline some ideas to do theoretical and numerical analysis for these problems. And, it is expected that solving these mathematical problems will help to improve the yield and promote industrialization of inkjet printing of advanced display films. On the other hand, we hope that this will provides mathematicians some new and interesting problems coming from inkjet printing industry. Also, we intend to provide the materialists in inkjet printing a mathematical route to

solve the key problems in experiments and productions. This is a good bridge of communication between mathematicians and materialists. This talk is based on the work joint with Zhouping Xin, Xiaoping Wang, Tiezheng Qian, Jinkai Li and Xinpeng Xu, and it is supported by the Key Project of the National Natural Science Foundation of China under grant 12131010.

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Well-Posedness and Low Mach Number Limit of the Free Boundary Problem for the EulerFourier System

Professor Xumin GU
Shanghai University of Finance & Economics

We consider the free boundary problem for the EulerFourier system that describes the motion of compressible, inviscid and heat-conducting fluids. The effect of surface tension is neglected and there is no heat flux across the free boundary. We prove the local well-posedness of the problem in Lagrangian coordinates under the Taylor sign condition. The solution is produced as the limit of solutions to a sequence of tangentially-smoothed approximate problems, where the so-called corrector is crucially introduced beforehand in the temperature equation so that the approximate initial data satisfying the corresponding compatibility conditions can be constructed. To overcome the strong coupling effect between the Euler part and the Fourier part in solving the linearized approximate problem, the temperature equation is further regularized by a pseudo-parabolic equation. Moreover, we prove the uniform estimates with respect to the Mach number of the solutions to the free-boundary EulerFourier system with large temperature variations, which allow us to justify the convergence towards the free-boundary inviscid low Mach number limit system by the strong compactness argument.

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Local well-posedness of heat conductive compressible Navier-Stokes equations in the presence of vacuum without compatibility conditions

Professor Jinkai LI
South China Normal University

In this talk, we consider the initial-boundary value problem to the heat conductive compressible Navier-Stokes equations. Local existence and uniqueness of strong solutions will be presented for any such initial data that the initial density ρ_0, velocity u_0, and temperature θ_0 satisfy $\rho_0 \in W^{1,q}$, with $q \in (3,6)$, $u_0 \in H^1$, and $\sqrt{\rho_0}\theta_0 \in L^2$. The initial density is assumed to be only nonnegative and thus the initial vacuum is allowed. In addition to the necessary regularity assumptions, we do not require any initial compatibility conditions such as those proposed by Cho and Kim, which although are widely used in many previous works but put some inconvenient constraints on the initial data. Due to the weaker regularities of the initial data and the absence of the initial compatibility conditions, leading to weaker regularities of the solutions compared with those in the previous works, the uniqueness of solutions obtained in this talk does not follow from the arguments used in the existing literatures. Our proof of the uniqueness of solutions is based on the following new idea of two-stages argument: (i) showing that the difference of two solutions (or part of their components) with the same initial data is controlled by some power function of the time variable; (ii) carrying out some singular-in-time weighted energy differential inequalities fulfilling the structure of the Grönwall inequality. The existence is established in the Euler coordinates, while the uniqueness is proved in the Lagrangian coordinates first and then transformed back to the Euler coordinates.

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On the global existence or blowup of weak solutions to the semilinear wave equations with time-dependent dissipation

Professor Huicheng YIN
Nanjing University

In this talk, we are concerned with the global existence or blowup of weak solutions to the n-dimensional semilinear wave equation with time-dependent scale-invariant damping. This equation can be transformed into the semilinear generalized Tricomi equation. Through determining the critical exponent $p_{crit} > 1$ and the conformal exponent $p_{conf} > p_{crit}$, and establishing the time-weighted Strichartz inequalities, the corresponding global existence or blowup results are obtained. This is a joint work with Dr. He Daoyin and Li Qianqian.

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Small Solitons and Multi-Solitons in Nonlinear Schrodinger Equations

Professor Jian ZHANG
University of Electronic Science and Technology of China

By introducing and solving two correlative constrained variational problems, a one-to-one correspondence from the prescribed mass to frequency of soliton is established for the double power nonlinear Schrodinger equation. The uniqueness of the normalized ground state is shown. Then orbital stability of small solitons depending on frequencies is proved. Moreover, multi-solitons with different speeds are constructed by stable small solitons.

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Incompressible limit of three-dimensional isentropic compressible Navier-Stokes equations with discontinuous initial data

Professor Xin ZHONG
Southwest University

In this talk, we consider the global weak solutions to the Cauchy problem of isentropic compressible Navier-Stokes equations in R3 with bounded initial density that are of small energy but possibly large oscillations with non-vacuum constant state as far field. These solutions converge globally in time to a global weak solution of the inhomogeneous incompressible Navier-Stokes equations as the bulk viscosity goes to infinity. This result generalizes Danchin-Mucha's works (Adv. Math. 320: 904-925, 2017 and Comm. Pure Appl. Math. 76: 3437-3492, 2023) on the incompressible limit for strong solutions to weak solutions that have discontinuous density along surfaces. Some new techniques based on the effective viscous flux are developed in order to obtain the uniform a priori estimates. This is a joint work with Professors Xianpeng Hu and Guochun Wu.