Local well-posedness of heat conductive compressible Navier-Stokes equations in the presence of vacuum without compatibility conditions

Professor Jinkai LI
South China Normal University

In this talk, we consider the initial-boundary value problem to the heat conductive compressible Navier-Stokes equations. Local existence and uniqueness of strong solutions will be presented for any such initial data that the initial density ρ_0, velocity u_0, and temperature θ_0 satisfy $\rho_0 \in W^{1,q}$, with $q \in (3, 6)$, $u_0 \in H^1$, and $\sqrt{\rho_0}\theta_0 \in L^2$. The initial density is assumed to be only nonnegative and thus the initial vacuum is allowed. In addition to the necessary regularity assumptions, we do not require any initial compatibility conditions such as those proposed by Cho and Kim, which although are widely used in many previous works but put some inconvenient constraints on the initial data. Due to the weaker regularities of the initial data and the absence of the initial compatibility conditions, leading to weaker regularities of the solutions compared with those in the previous works, the uniqueness of solutions obtained in this talk does not follow from the arguments used in the existing literatures. Our proof of the uniqueness of solutions is based on the following new idea of two-stages argument: (i) showing that the difference of two solutions (or part of their components) with the same initial data is controlled by some power function of the time variable; (ii) carrying out some singular-in-time weighted energy differential inequalities fulfilling the structure of the Grönwall inequality. The existence is established in the Euler coordinates, while the uniqueness is proved in the Lagrangian coordinates first and then transformed back to the Euler coordinates.