

IMS Workshop on PDEs (I)

(January 28-30, 2026)

TITLE & ABSTRACT

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**Steady compressible Navier-Stokes-Fourier system with slip boundary conditions arising from kinetic theory**

**Professor Renjun DUAN**  
*The Chinese University of Hong Kong*

This talk concerns the boundary value problem on the steady compressible Navier-Stokes-Fourier system in a channel domain  $(0, 1) \times \mathbb{T}^2$  with a class of generalized slip boundary conditions that were systematically derived from the Boltzmann equation by Coron [JSP, 1989] and later by Aoki et al. [JSP, 2017]. We establish the existence and uniqueness of strong solutions in  $(L^2_0 \cap H^2(\Omega)) \times V^3(\Omega) \times H^3(\Omega)$  provided that the wall temperature is near a positive constant. The proof relies on the construction of a new variational formulation for the corresponding linearized problem and employs a fixed point argument. The main difficulty arises from the interplay of velocity and temperature derivatives together with the effect of density dependence on the boundary.

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**On the Forward Self-Similar Solutions to the Two-Dimensional Navier-Stokes Equations**

**Professor Changfeng GUI**  
*Zhuhai UM Science and Technology Research Institute, University of Macau*

I will present a recent result on the global existence of forward self-similar solutions to the two-dimensional incompressible Navier-Stokes equations for any divergence-free initial velocity that is homogeneous of degree  $-1$  and locally Hölder continuous. This result requires no smallness assumption on the initial data. In sharp contrast to the three-dimensional case, where  $(-1)$ -homogeneous vector fields are locally square-integrable, the major difficulty for the 2D problem is the criticality in the sense that the initial kinetic energy is locally infinite at the origin, and the initial vorticity fails to be locally integrable, so that the classical local energy estimates are not available. Our key ideas are to decompose the solution into a linear part solving the heat equation and a finite-energy perturbation part, and to exploit a kind of inherent cancellation relation between the linear part and the perturbation part. These, together with suitable choices of multipliers, enable us to control the interaction terms and to establish the  $H^1$ -estimates for the perturbation part. Furthermore, we can get an optimal pointwise estimate via investigating the corresponding Leray equations in weighted Sobolev spaces. This gives the faster decay of the perturbation part at infinity and compactness, which play important roles in proving the existence of global-in-time self-similar solutions.

This talk is based on a joint work with Hao Liu and Chunjing Xie.

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**The stabilizing effect of gravity on viscous compressible stratified fluids without surface tension in horizontally infinite domains**

**Professor Zhenhua GUO**  
*Guangxi University*

In this talk, we will investigate the motion of barotropic, viscous, compressible fluids with a free interface separating two fluid flows. In this free-boundary problem, the stratified fluids occupy a horizontally infinite three-dimensional domain bounded below by a fixed bottom and above by a moving surface under constant pressure, and gravity is taken into account while surface tension effects are ignored. By fully exploring the energy-dissipation structure, we establish the global well-posedness of this problem based on the Lagrangian framework in relatively low-regularity Sobolev spaces. Furthermore, we reveal that the absence of the Rayleigh–Taylor instability condition has an essential stabilizing effect on the model and that the vertical velocity decays to zero at a time-integrable rate that is faster than that of the horizontal velocity under gravity, which conforms to physical intuition. The key insight of our proof is that the commutator estimates between Lagrangian temporal and Eulerian spatial derivatives are sharpened via the faster decay rates of the vertical velocity, which enhances the energy estimates for the acceleration system in its geometric form. We also construct a series of a priori estimates of the quantities at the internal interface including the pressure difference. As a by-product of our proof, we develop a novel set of techniques applicable to certain other free-boundary problems of two-phase flows. This work is joint with Dr. Yuhong Gao and Lei Xu.

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**Global existence and decay rates for the 3D compressible flows with eddy diffusion**

**Professor Guangyi HONG**  
*South China University of Technology*

In this talk, we will introduce our results on the Cauchy problem of the 3D compressible Navier-Stokes equations with eddy diffusion which is commonly used in the study of geophysical flows (cf. Jabin-Bresch, *Ann. of Math.* 2018). The prominent character of the model is the lack of vertical dissipation of the velocity field. The nonlinear asymptotic stability of the constant equilibrium state with strictly positive constant density and vanishing velocity is established under suitable small initial perturbation in regular Sobolev spaces. Specifically, we show the convergence of the density and velocity towards the corresponding equilibrium state in  $H^2(\mathbb{R}^3)$  with almost optimal decay rates. The proof is based on the detailed analysis of the Green function, the time-weighted energy estimates, and the intrinsic coupling structure of the system. As an application, we will also introduce our recent results on the 3D compressible MHD equations with eddy diffusion for the velocity field and horizontal dissipation for the magnetic field.

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**Concentration of weak solutions in compressible flows**

**Professor Xianpeng HU**  
*The Hong Kong Polytechnic University*

We will discuss some recent progress in the concentration phenomenon of weak solutions for compressible flows. We will focus on two typical examples, compressible viscoelasticity and compressible Navier-Stokes equations. Some new observations will be discussed.

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**Some recent progress on free boundary value problems for the Boltzmann equation**

**Professor Shuangqian LIU**  
*Central China Normal University*

In this talk, I will report on our recent work on a free boundary value problem for the Boltzmann equation, where the boundary motion is governed by an ordinary differential system derived from Hooke’s law and is coupled with the drag force exerted by the surrounding particles. A central difficulty of this problem lies in the strong interaction between the kinetic dynamics and the free boundary. To address this issue, we introduce a novel conformal transformation that reduces the moving-domain problem to a fixed reference domain. A key observation is that the kinetic distribution and the free boundary variables, when considered jointly, generate a natural quadratic energy structure that reveals intrinsic damping and cancellation effects at the boundary. Exploiting this structure, we establish global-in-time existence of solutions via an  $L^\infty - L^2$  framework. Furthermore, the uniqueness of solutions is obtained by proving localized weighted  $W^{1,p}$  estimate, which yield stability in  $L^{1+}$ .

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**Some PDE Models from Ecology**

**Professor Yuan LOU**  
*Shanghai Jiao Tong University*

Understanding the structure, function, and interconnections of biological systems is an intriguing question. In ecosystems, how does population dispersal affect the system’s structure and stability? We will analyze multiple partial differential equation models to explore the impact of spatial dispersal and environmental factors on population distribution (corresponding to structure). Then, using game theory, we will reveal various evolutionarily stable dispersal strategies (corresponding to function), and finally provide intuitive biological interpretations through ideal free distribution theory (linking structure and function).

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**A Class of Global Solutions to 3D Free-Boundary Relativistic Euler Equations with a Physical Vacuum Boundary**

**Professor Chenyun LUO**  
*The Chinese University of Hong Kong*

We consider the free-boundary relativistic Euler equations in Minkowski spacetime equipped with a physical vacuum boundary, which models the motion of a relativistic gas. We concern ourselves with the family of isentropic, barotropic, and polytropic gas, with an equation of state  $p = \rho^{1+\kappa}$ ,  $\kappa \in (0, 2/3]$ . We construct an open class of initial data that launches future-global solutions. Such solutions are spherically symmetric, have small initial density, and expand asymptotically linearly in time. In particular, the asymptotic rate of expansion is allowed to be arbitrarily close to the speed of light. Therefore, our main result is far from a perturbation of existing results concerning the classical isentropic Euler counterparts. This is a joint work with Marcelo M. Disconzi (Vanderbilt) and Zhongtian Hu (Princeton).

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**On vacuum boundary with physical singularity for compressible Euler equations – old and new**

**Professor Tao LUO**  
*City University of Hong Kong*

In this talk, I will first review the background, challenge and known results in the study of physical vacuum of compressible inviscid fluids. Then I will discuss the problem of global in time behavior for the system with damping including the old joint work with Prof Huihui Zeng and some recent developments. Lastly, I will present the recent joint work with Dr. Sicheng Liu on the local well-posedness in low-regularity weighted spaces in Hadamard sense for full compressible Euler equations in multi-dimensions.

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**Classification of global dynamics of a periodic diffusive consumer-resource model**

**Professor Zhi-An WANG**  
*Hong Kong Polytechnic University*

This talk is concerned with a consumer-resource system, which incorporates resource decay and consumer loss, under conditions where the resource input rate may exhibit spatial heterogeneity and temporal periodicity. We characterize the persistence or extinction of both consumer and resource populations based on their dispersal rates and a relaxation-time parameter. Additionally, we derive the asymptotic profiles of positive periodic solutions as the resource dispersal rate becomes sufficiently small or large. Our results reveal several notable insights: (a) resource decay, even when slight, acts as a decisive factor preventing unlimited growth of resource abundance; (b) the consumer mortality rate is a key determinant of the consumer population’s persistence or extinction; (c) when the consumer mortality rate is moderate, a temporally homogeneous resource input may be more advantageous for the consumer population than a temporally periodic one.

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**On Brezis’ two open questions**

**Professor Juncheng WEI**  
*The Chinese University of Hong Kong*

In 2023, one year before he died, H. Brezis proposed 28 open questions that “raised throughout his career and that have resisted so far”. In this talk, we give a complete solution to the first open problem (Problem 1.1 on Brezis-Nirenberg problem), and a partial solution to the 7th open problem (Problem 3.1 on harmonic maps).

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**Regular Shock Reflection-Diffraction Problem**

**Professor Wei XIANG**  
*City University of Hong Kong*

We will talk about our recent results on the convexity, uniqueness, and stability of regular reflection solutions for the potential flow equation in a natural class of self-similar solutions. The approach is based on a nonlinear version of the method of continuity. Finally, our recent work on the isentropic Euler equations will be discussed, too.

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**Shock formation of 3-D supersonic steady flows for the squeezed short pulse initial data**

**Professor Huicheng YIN**

*Nanjing University*

In this talk, we are concerned with the shock formation problem for the 3-D steady supersonic potential equation of polytropic gases. For a class of short pulse initial data, as long as the initial supersonic flow is strongly squeezed at some point, it is shown that the shock will be formed in finite distance without any other smallness restrictions. One of our main ingredients is to find a new good unknown so that the inverse foliation density  $\mu \rightarrow 0$  can be shown. This is a joint work with Prof. Xin Zhouping and Prof. Ding Bingbing.