East-Asia Workshop on Pattern Formations and Reaction-Diffusion Systems

Program

All lectures will take place at Academic Bulding No.1, Room 501a, Institute of Mathematical Sciences, Chinese University of Hong Kong

Thursday–Dec. 14, 2006

9:40-10:10am Tea Break and Registration

10:10-11:00am Professor Y. Nishiura (RIES, Hokkaido University, Japan)
Title:*Heterogenitiy-induced pulse dynamics in dissipative systems*

Abstract

Particle-like (spatially localized) dissipative patterns arise in many fields such as chemical reaction, gas-discharge system, liquid crystal, binary convection, and morphogenesis. We discuss about the dynamics of moving particles in heterogeneous media, especially for the three types of heterogeneities: jump, bump, and periodic media respectively. Model systems include the Gray-Soctt model and a three-component reaction diffusion system of one-activator-two-inhibitor type. We focus on the following issues:(a) heterogeneity-induced defect-structure created around the jump point, (b) collision dynamics between particles and defects, (c) unstable objects called scattors which sort out the destinations of orbits, and (d) an organizing center creating traveling pulses, defects, and scattors via unfolding. When traveling objects like pulses or spots encounter heterogeneities, a variety of outputs are produced such as annihilation, rebound, splitting, and relaxing to an ordered pattern. It turns out that there is a class of unstable patterns called scattors which play a role of separator whose unstable manifolds guide the orbits to their destinations. For 1D case, a unified view is presented by a global solution branch with respect to the parameters chracterizing the heterogeneities that contain all the above important objects including traveling pulses, defects, and scattors.

11:20am-12:00noon Professor Kei-Ichi Ueda (Kyoto University, Japan)
Title:*Scattering dynamics of traveling pulses in reaction-diffusion systems*

Abstract

Scattering process between traveling pulses in a three-component reaction diffusion system is studied. We focus on the issue how the input-output relation is controlled at a head-on collision in the one-dimensional space where traveling pulses interact strongly. We found that a hidden saddle plays a crucial role to understand the scattering dynamics and their unstable manifolds direct the traffic flow of orbits.

12:00-2:00pm Lunch Time

2:00-2:50pm Professor M. J. Ward (UBC, Canada)

Title: *A Two-Dimensional Metastable Flame-Front and a Degenerate Spike-Layer Problem*

Authors: Alexei Cheviakov (Postdoctoral Fellow, UBC); Michael J. Ward (UBC)

Abstract

A formal asymptotic analysis is used to analyze the metastable behavior associated with a nonlocal PDE model describing the upward propagation of a flame-front interface in a vertical channel with a two-dimensional convex cross-section. In a certain asymptotic limit, the flame-front interface assumes a roughly paraboloidal shape whereby the tip of the paraboloid drifts asymptotically exponentially slowly towards the closest point on the wall of the channel. Asymptotic estimates for the exponentially small eigenvalues responsible for this metastable behavior are derived together with an explicit ODE for the slow motion of the tip of the paraboloid. The subsequent slow motion of the tip along the channel wall is also characterized explicitly. The analysis is based on a nonlinear transformation that has the effect of transforming the paraboloidal interface to a spike-layer solution of a specific singularly perturbed quasilinear parabolic problem with a non-differentiable quasilinear term.

3:00am-3:40pm Professor Masaharu Nagayama (Kanazawa University, Japan)
Title : *Mathematical modeling for the motion of oil droprate*

3:40pm-4:10pm Tea Break

4:10pm-4:50pm Professor Seiro Omata (Kanazawa University, Japan)
Title: *Bubble motion on water surface*

Abstract

The motion of a bubble on water surface is treated numerically. The bubble is assumed to keep its volume and the edge of the bubble repr esents free boundary. The problem becomes a free boundary problem of degenerate hyperbolic type with volume conservation. The notion of a weak solution to this problem is introduced and justified. A minimizing method using the discrete Morse flow of hyperbolic type works well in the numerical solution of this problem.

5:00pm-5:40pm Professor Yaping Wu (Capital Normal University, China)

Title: *The Stability of Travelling Waves with Algebraic Decay for Some Autocatalytic Reaction Systems*

Authors: Yi Li, Yaping Wu, Xiuxia Xing, Qixiao Ye.

Abstract

Consider the following reaction-diffusion systems arising in autocatalytic reaction models

$$\begin{cases} u_t = du_{xx} - u^p v^q, \\ v_t = v_{xx} + u^p v^q, \end{cases}$$

For each fixed $p \geq 1$, $q \geq 1$, and $d \geq 0$, there exist a critical speed $c^*(p,q,d)$ such that for any $c \geq c^*(p,q,d)$ there exist travelling front solutions $(u(x-ct), v(x-ct))$ connecting $(0,1)$ and $(1,0)$. For the cases $p > 1$ or $q > 1$, the travelling waves with noncritical speed decay algebraically in space at $-\infty$ or $+\infty$.

Here we are only interested in the case $p > 1$ or $q > 1$.

For $d = 1$, the system can be reduced to the so-called p-degree Fisher equation

$$u_t = u_x x + u^p(1-u), \;\; p > 1,$$

or double degenerate Fisher equation

$$u_t = u_x x + u^p(1-u)^q, \;\; p > 1, q > 1.$$

By Evan's function method, detailed spectral analysis and semigroup decaying estimates, we can prove that all the travelling fronts of the p-degree Fisher equation and double degenerate Fisher are asymptotically stable in some polynomially weighted spaces, and the travelling front with critical speed for p-degree Fisher equation is globally stable.

For the cases d is near 1, $p > 1$ and $q = 1$, by Evan's function method and detailed spectral analysis , the travelling waves with algebraic spacial decay can be proved to be exponentially stable in some exponentially weighted spaces.

6:40pm Reception

Friday—Dec. 15, 2006

9:10am-10:00 am Professor Xiaofeng Ren (Utah State University, USA)

Title: *Many droplet pattern in the cylindrical phase of diblock copolymer morphology*

Abstract

The Ohta-Kawasaki density functional theory of diblock copolymers gives rise to a nonlocal free boundary problem. In a proper parameter range an equilibrium pattern of many droplets is proved to exist in a general planar domain. A sub-range is identified where the multiple droplet pattern is stable. Each droplet is close to a round disc. The boundaries of the droplets satisfy an equation that involves the curvature of the boundary and a quantity that depends nonlocally on the whole pattern. The locations of the droplets are determined via a Green's function of the domain. In constructing the droplet pattern we overcome three obstacles: interface oscillation, droplet coarsening, and droplet translation.

10:00am-10:30am Tea Break

10:30am-11:10am Professor Shigetoshi Yazaki (University of Miyazaki, Japan)

Title: *Crystalline curvature flow equations I— mathematical aspects and a modelling perspective*

Abstract

Motion by curvature is generally referred as a motion of curves in the plane (or surfaces in the three dimensional space) which changes its shape in time and depend on its bend, especially on its curvature (in higher dimensional space, mean curvature, Gauss curvature, and so on). They are also called curvature flows, since one tracks flows of the family of curves (or surfaces) parameterized in time. The curvature flow equation is a general term which describes such flows, and has been investigated by many material scientists, physicians and mathematicians since the 1950's. The crystalline curvature flow equation has been appeared at the end of 1980's by J. E. Taylor, and S. Angenent and M. E. Gurtin.

Crystalline curvature flow is a flow of polygonal curves or surfaces in a special class. It can be regarded as a discrete version of smooth curvature flow from numerical points of view. Therefore, sometimes properties of solutions of crystalline curvature flow are similar to the one in the smooth case. However, sometimes they are strikingly different from the smooth case. This point is one of attractive interests of concerning crystalline curvature flow . In this series of two talks, we will see overview of crystalline curvature flow, especially in Part I, from mathematical and physical points of view, and in Part II, from numerical points of view.

The crystalline curvature for polygonal curves (or surfaces) is an analogue of the usual curvature for smooth curves (or surfaces), and the crystalline curvature flow equations are the equations which describe the motion of curves (or surfaces) whose normal velocity V depends on crystalline curvature κ. Such notion were introduced by J. E. Taylor, and S. Angenent and M. E. Gurtin, independently, at the end of 1980's.

In this talk, first, we will summarize the mathematical background of crystalline curvature flow equations and the known results, mostly in the case where the space dimension is two.

Second, we will show our several mathematical results. Comparing with the case of the curvature flow equations (the case of smooth curves), mathematical results for the crystalline curvature flow equations are not so clarified. For example, the convexification theorem is well known for the curvature flow equation $V = \kappa^{\alpha}(\alpha > 0)$, namely, the smooth closed curve, which is initially non-convex, eventually becomes convex and then disappears in finite time by this evolution. However, such property does not always hold for the corresponding crystalline curvature flow equations. We will explain a result of similar property for these equations. We will also show that the asymptotic behavior of an area-preserving crystalline curvature equations.

Finally, we will mention an application of the crystalline curvature equations to the modelling of some physical phenomena.

11:20am-12:00noon Professor Takeo K. Ushijima (Tokyo University of Science, Japan)

Title: *Crystalline curvature flow equations II— numerical computations in 2D and 3D*

Abstract

Crystalline curvature flow equations can be used as an approximation of the corresponding curvature flow equations. Such a way of

approximation is called crystalline algorithm. In this talk, we will explain several applications of the crystalline algorithm to the numerical computations of curvature dependent motions: We will mainly focus on the evolution equation $V = \kappa^\alpha$ and an area-preserving curvature flow equation in the case where space dimension is two, and the Gauss curvature flow equation in the case where space dimension is three.

The relation between crystalline curvature flow equations and the corresponding curvature flow equations has been studied extensively by several authors. We will show several convergence results. Among of them, the convergence result for the Gauss curvature flow, which is recently obtained by us, will be explained precisely.

Next, a numerical scheme which is constructed using crystalline algorithm will be introduced. This numerical scheme is a kind of direct method for moving boundary problems. Although such direct methods often cause numerical instability, our method does not show such instability without using any artificial technique. The method based on crystalline algorithm will be compared with the other direct methods, especially the method by using intrinsic heat equation. The reason of this good numerical stability of our method will be considered.

We will also show several numerical examples.

12:00noon-2:00pm Lunch

2:00-2:50pm Professor Tai-Chia Lin (National Taiwan University)

Title: *Bending-wave instability of a vortex ring in a trapped Bose-Einstein condensate*

Abstract

Using a velocity formula derived by asymptotic expansion, we study the dynamics of a vortex ring in an axisymmetric Bose-Einstein condensate in the Thomas-Fermi limit. The trajectory for an axisymmetrically placed and oriented vortex ring shows that it generally precesses in a condensate. The linear instability due to bending waves is investigated both numerically and analytically. General stability boundaries for various perturbed wave numbers are computed. Our analysis suggests that a slightly oblate trap is needed to prevent the vortex ring from becoming unstable.

2:50-3:20pm Tea Break

3:20pm-4:00pm Professor Theodore Kolokolnikov (Dalhousie University, Canada)
Title: *Mesa-splitting in Reaction-Diffusion Systems*

Abstract

We consider several reaction-diffusion systems that exhibit mesa (box-like) patterns. These include the Brusselator, the Lengyel-Epstein model, and the Gierer-Meinhardt model with saturation. We then show that these systems exhibit self-replication (nucleation) as the control parameter is varied. Near the self-replication threshold, we show that a boundary layer is formed. The solution within this layer is governed by a universal *core problem*, a single non-autonomous second order ODE which is the same for all the models considered. By analysing this ODE, we rigorously show that the conditions for self-replication are satisfied.

This is a joint work with Juncheng Wei and Michael Ward.

4:00-4:10pm Break

4:10pm-4:50pm Professor Juncheng Wei (Chinese University of Hong Kong)
Title: *Mutually Exclusive Spiky Pattern and Segmentation Modelled by the Five-Component Meinhardt-Gierer System*

Abstract

We consider the five-component Meinhardt-Gierer model for mutually exclusive patterns and segmentation which was proposed by Meinhardt in his book "*Boiological Pattern Formations*". We prove rigorous results on the existence and stability of mutually exclusive spikes which are located in different positions for the two activators.

Sufficient conditions for existence and stability are derived, which depend in particular on the relative size of the various diffusion constants. Our main analytical methods are the Liapunov-Schmidt reduction and nonlocal eigenvalue problems. The analytical results are confirmed by numerical simulations.