



IMS Workshop on Partial Differential Equations from Fluid Dynamics

February 5, 2008

Title and Abstract

Decay Properties for Solutions to the Navier-Stokes Equations

Cheng He

National Natural Science Foundation of China & IMS, CUHK

Abstract

In this talk, we will present some basic results on the asymptotic behaviour for solutions to the incompressible Navier-Stokes equations. We will mainly analyze the significant difference of the asymptotic behaviour of solutions between the whole space and the domain with non-empty boundary. Finally, we will present some recent results on the decay properties of solutions to the Navier-Stokes equations in the half space.

Bound States of Nonlinear Schrödinger Equations with Potentials Tending to Zero at Infinity

Huicheng Yin

Nanjing University & IMS, CUHK

(This is a joint work with Dr. Pingzheng Zhang)

Abstract

In this paper, we are concerned with the existence of solutions to the N -dimensional nonlinear Schrödinger equation $-\varepsilon^2 \Delta u + V(x)u = K(x)u^p$ with $u(x) > 0$, $u \in H^1(\mathbb{R}^N)$, $N \geq 3$ and $1 < p < \frac{N+2}{N-2}$. When the potential $V(x)$ decays at infinity faster than $(1 + |x|)^{-2}$ and $K(x) \geq 0$ is permitted to be unbounded, we will show that the positive $H^1(\mathbb{R}^N)$ -solutions exist if it is assumed that $G(x)$ has local minimum points for small $\varepsilon > 0$, here $G(x) = V^\theta(x)K^{-\frac{2}{p-1}}(x)$ with $\theta = \frac{p+1}{p-1} - \frac{N}{2}$ denotes by the ground energy function. In addition, when the potential $V(x)$ decays to zero at most like $(1 + |x|)^{-\alpha}$ with $0 < \alpha \leq 2$, we also discuss the existence of positive $H^1(\mathbb{R}^N)$ -solutions for unbounded $K(x)$. Compared with some previous papers, we remove the restrictions on the potential function $V(x)$ which decays at infinity like $(1 + |x|)^{-\alpha}$ with $0 < \alpha \leq 2$ as well as the restrictions on the boundedness of $K(x) > 0$. Therefore, we partly answer an open question posed in Ambrosetti's paper.

Diffusive Limit for the Boltzmann Equation in the Whole Space

Huijiang Zhao
Wuhan University & CityU

Decay Rates to the Planar Rarefaction Waves for a Model System of the Radiating Gas in n -dimensions

Changjiang Zhu
Central China Normal University & CityU

Abstract

In this paper, we study the asymptotic decay rates to the planar rarefaction waves to the Cauchy problem for a hyperbolic-elliptic coupled system called as a model system of the radiating gas in \mathbb{R}^n ($n = 3, 4, 5$) if the initial perturbations corresponding to the planar rarefaction waves are sufficiently small in $(H^2 \cap L^1 \cap W^{2,6})(\mathbb{R}^n)$. The analysis is based on the L^p -energy method and several special interpolation inequalities.

This is a joint work with Wenliang Gao and Lizhi Ruan.

Heat Flow for the Square Root of the Negative Laplacian for Unit Length Vectors

Zheng-An Yao

Zhongshan University & IMS, CUHK

Abstract

The paper is concerned with heat flow for the square root of the negative Laplacian for unit length vectors, motivated by the model of DeSimone, Kohn, Otto and Müller. We show the existence of weak solutions for the periodic case by means of the penalty approximation and commutator estimate.

Subsonic and Subsonic-sonic Flows in Nozzles

Chunjing Xie

IMS, CUHK

Abstract

In this talk, I will review some recent progress on subsonic and subsonic-sonic flows in nozzles, where both potential flows and Euler flows will be discussed.

One-side Mixed Initial-boundary Value Problem for Quasilinear Hyperbolic Systems

Libin Wang

Fudan University & IMS, CUHK

Abstract

In this paper, we consider the mixed initial-boundary value problem for quasilinear hyperbolic systems with nonlinear boundary conditions in a half-unbounded domain $\{(t, x) \mid t \geq 0, x \geq 0\}$. Under the assumption that the positive eigenvalues are weakly linearly degenerate, we obtain the global existence and uniqueness of C^1 solution with small and decaying initial data. Some applications are given for the system of the planar motion of an elastic string.

Support Properties of Solution to Degenerate Parabolic Equation with Variable Coefficient and Absorption

Chunlai Mu

Sichuan University & CityU

Abstract

In this talk, we deal with the properties of solution to Cauchy problem for the degenerate parabolic equation with variable coefficient and absorption (convection). We firstly give a critical exponent which distinguishes the localization of solutions from the positivity of them. Secondly, we obtain another critical exponent with respect to the decay behavior of the coefficient at infinity, which separates the global existence of interfaces from the disappearance of interfaces in finite time. Finally, we show that instantaneous shrinking of the support of the solutions depends on the behavior of variable coefficient.

The Global Behavior of Entropy Solutions to the Relativistic Euler Equations

Ya-Chun Li

Shanghai Jiao Tong University & CityU

Abstract

In this talk, we will discuss some of the recent developments in the systematic study of global entropy solutions to the relativistic Euler equations.

Stability of Boundary Layer and Rarefaction Wave to an Outflow Problem for Compressible Navier-Stokes Equations with Large Perturbation

Feimin Huang

Chinese Academy of Sciences & CityU

A Note on the Positive Solutions of an Inhomogeneous Elliptic Equation on \mathbb{R}^n

Yinbin Deng

Huazhong Normal University & CityU

(Joint with Yi Li and Fen Yang)

Abstract

This paper is contributed to the elliptic equation

$$\Delta u + K(|x|)u^p + \mu f(|x|) = 0 \quad (0.1)$$

where $p > 1, x \in \mathbb{R}^n, n \geq 3, \Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ and $\mu \geq 0$ is a constant. We study the structure of positive radial solutions of (0.1) and obtain the uniqueness of solution decaying faster than r^{-m} at ∞ if μ is small enough under some assumptions on K and f , where m is the slow decay rate.

The Initial Boundary Value Problem for Multi-Dimensional Euler Equation with Damping

Weike Wang

Shanghai Jiao Tong University & CityU

(Joint work with ShiJin Deng and Yongqin Liu)

Abstract

In this talk, we discuss the initial boundary value problem (IBVP) for the multi-dimensional euler equation with damping term by posing two different physical boundaries. For the case with boundary $x = bt(b > 0)$, we get the global-in-time existence of classical solution to the initial-boundary value problem by the method of energy estimates estimate, and the decays exponentially by weighted energy estimate. The case with boundary $x = bt(b < 0)$ is totally different. The solution doesn't possess exponential decaying rate and the energy estimate will not yield a decay estimate. Based on the Green's function for the Cauchy problem constructed in [1] and the boundary energy estimate, we gain the large-time behavior of the solution.

Reference

- [1] Weike Wang and Tong Yang, The Pointwise Estimates of Solutions for Euler Equations with Damping in Multi-Dimensions. *Journal of Differential Equations*. 173 (2001), 410–450.