Periodic Bifurcation and Soliton deflexion for Kadomtsev-Petviashvili Equation

*Zhengde Dai**

(*Department of Information and Computing Science, Guangxi Industrial College Liuzhou, Guangxi, 545005, P.R.China*)

(*The Institute of Mathematical Sciences,The Chinese University of Hong Kong*)

(*School of Network,Yunnan University,Kunming,650091,P.R.China*)

Shaolin Li

(*Department of Mathematics, Honghe College, Mengzi, Yunnan, 661100, P.R.China*)

Aijun Zhu

(*School of Mathematics and Physics,Nanhua University, Hengyang, Hunan, 421001, P.R.China*)

Abstract The exact two-soliton solution to KPI and a class of periodic soliton solutions to KPII are obtained by using bilinear form and variable translation. At the same time, making a temporal and spatial transformation induces the doubly periodic solution to KPI and another class of periodic soliton solution to KPII. It is also investigated that the equilibrium solution $u_0 = -\frac{1}{6}$ is an unique bifurcation which is periodic bifurcation for KPI and deflexion of soliton for KPII .

Key Words periodic bifurcation, soliton deflexion,Kadomtsev-Petviashvili Equation

1.Introduction

It is well established now that the Kadomtsv-Petviashvili(KP) equation is the key ingredient in a number of remarkable nonlinear problems, both in physics and mathematics [1,2]. It was derived initially to examine small effects in a direction perpendicular to the propagation direction would have on a KdV soliton in a plasma[3]. It is only weakly dependent on the transverse coordinate and has been frequently stated to apply to one and one-half dimensions.

The solutions of KP equation have been extensively studied since they were first found. Various methods had been tried and many special solutions were given[4-9].A prominent feature of the KP equation is that it admits an interaction and resonance of several individual solitons resulting in a drastic increase of the amplitude if the solitons in the interaction region[10-12].

In this paper, several types of exact solutions to KP equation were constructed by bilinear form and variable translation. It is explicitly analyzed that the feature of the solutions is different on the both sides of equilibrium solution $u_0 = -\frac{1}{6}$, which is a unique periodic bifurcation point for KPI and deflexion point of soliton for KPII . As for KPI, when the equilibrium u_0 varies from one side of $-\frac{1}{6}$ to another side, two-soliton solution changes into doubly periodic solution. Whereas, the y-periodic soliton changes into $x - t$-periodic soliton for KPII.

[0*]) Corresponding author.Tel/fax: +86 871 503 1603.

[1]) E-mail addresses: zhddai2004@yahoo.com.cn; shaolinli@163.com; ajzhu0215@163.com

[2])This work was supported by Chinese Natural Science Foundation Grant No.10361007 and Yunnan Natural Science Foundation Grant No.2004A0001M

2. The two-solitons to KPI and periodic soltions to KPII

The Kadomtsev-Petviashvili equation in normalised varilable (u,x,y,t) reads

$$u_{xt}-u_{xxxx}-3(u^2)_{xx}-p^2u_{yy}=0 \tag{1}$$

where $u:R_x\times R_y\times R_t\to R$. Eq(1) is called KPI when $p^2=1$, and KPII when $p^2=-1$.It is easily to note that $u=u_0$ is an equilibrium solution of KP equation,where u_0 is an arbitrary constant.

Now consider KPI equation

$$u_{xt}-u_{xxxx}-3(u^2)_{xx}-u_{yy}=0 \tag{2}$$

Setting $\xi=i(x-t)$ in eq(2) gives

$$u_{yy}-u_{\xi\xi}-3(u^2)_{\xi\xi}+u_{\xi\xi\xi\xi}=0 \tag{3}$$

Let $u=u_0+2v,\ \ v=-(\ln F)_{\xi\xi}$, then (3) can be reduced into the following bilinear form

$$[D_y^2-(1+6u_0)D_\xi^2+D_\xi^4-A]F\cdot F=0 \tag{4}$$

where A is an integration constant and the Hirota bilinear operator $D_x^mD_t^k$ are defined by ref.[4]

$$D_x^mD_t^ka\cdot b=(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'})^m(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'})^ka(x,t)b(x',t)|_{x'=x,t'=t}$$

With regard to (4), we are going to seek the solution of the form

$$F=1+b_1(e^{ip\xi}+e^{-ip\xi})e^{\Omega y+\gamma}+b_2e^{2\Omega y+2\gamma} \tag{5}$$

where A, p, Ω, γ, b_1 and b_2 are all real.

Substituting (5) into (4) yields the exact solution of eq(3) of the form

$$u=u_0+\frac{2p^2[4b_1^2+b_1(e^{ip\xi}+e^{-ip\xi})(b_2e^{\Omega y+\gamma}+e^{-\Omega y-\gamma})]}{[b_1(e^{ip\xi}+e^{-ip\xi})+(b_2e^{\Omega y+\gamma}+e^{-\Omega y-\gamma})]^2} \tag{6}$$

where coefficients satisfy

$$A=0 \qquad \Omega^2=-p^4-(1+6u_0)p^2 \qquad b_1^2=\frac{\Omega^2b_2}{\Omega^2-3p^4} \tag{7}$$

It is obviously that $u_0<-\frac{1}{6}$ is required so that the conditions $\Omega^2>0$, $b_1^2>0$ and $p^2<-\frac{1+6u_0}{4}$ can be satisfied in eq(7),in this case u_0 is considered as a parameter.

Taking $\xi=i(x-t)$ into eq(6), the exact solution to KPI equation is expressed by

$$u_1(x,y,t)=u_0+\frac{2p^2[4b_1^2+b_1(e^{p(x-t)}+e^{-p(x-t)})(b_2e^{\Omega y+\gamma}+e^{-\Omega y-\gamma})]}{[b_1(e^{p(x-t)}+e^{-p(x-t)})+(b_2e^{\Omega y+\gamma}+e^{-\Omega y-\gamma})]^2} \tag{8}$$

where

$$\begin{cases} u_0<-\frac{1}{6} \\ p^2<-\frac{1+6u_0}{4} \\ \Omega^2=-p^4-(1+6u_0)p^2 \\ b_1^2=\frac{\Omega^2b_2}{\Omega^2-3p^4} \end{cases} \tag{9}$$

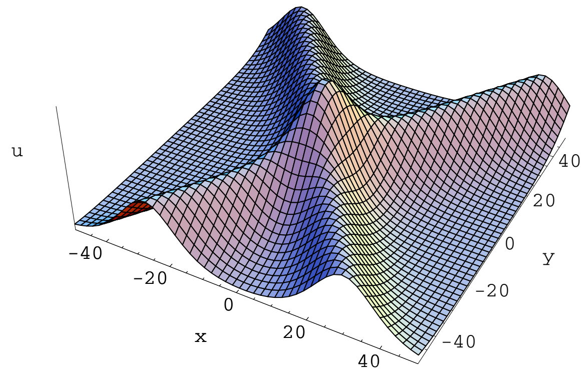


Figure 1: The two-soliton solution for KPI equation as $u_0 = -\frac{1}{4}$

According to expression eq(8) of u_1, the two-soliton solution to KPI is obtained. (ref Fig.1)

The KPII equation is given by

$$u_{xt} - u_{xxxx} - 3(u^2)_{xx} + u_{yy} = 0 \tag{10}$$

Making a variable transformation $\xi = x - t$, eq(10) can be transformed into the following form

$$u_{yy} - u_{\xi\xi} - 3(u^2)_{\xi\xi} - u_{\xi\xi\xi\xi} = 0 \tag{11}$$

Letting $u = u_0 + 2v, \;\; v = (\ln F)_{\xi\xi}$, being similar to the way of dealing with KPI, we take

$$F = 1 + b_1(e^{ip\xi} + e^{-ip\xi})e^{\Omega y+\gamma} + b_2 e^{2\Omega y+2\gamma}$$

By computing, the exact solution of (11) is given by

$$u = u_0 - \frac{2p^2[4b_1^2 + b_1(e^{ip\xi} + e^{-ip\xi})(b_2 e^{\Omega y+\gamma} + e^{-\Omega y-\gamma})]}{[b_1(e^{ip\xi} + e^{-ip\xi}) + (b_2 e^{\Omega y+\gamma} + e^{-\Omega y-\gamma})]^2} \tag{12}$$

where coefficients satisfy

$$\Omega^2 = p^4 - (1 + 6u_0)p^2 \qquad\qquad b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 + 3p^4} \tag{13}$$

It is easily to see that $u_0 \geq -\frac{1}{6}$ is available as long as $p^2 \geq 1 + 6u_0$.

Taking $\xi = x - t$ into eq(12), the exact solution to KPII is expressed

$$u_2(x, y, t) = u_0 - \frac{2p^2[4b_1^2 + b_1(e^{ip(x-t)} + e^{-ip(x-t)})(b_2 e^{\Omega y+\gamma} + e^{-\Omega y-\gamma})]}{[b_1(e^{ip(x-t)} + e^{-ip(x-t)}) + (b_2 e^{\Omega y+\gamma} + e^{-\Omega y-\gamma})]^2} \tag{14}$$

where

$$\begin{cases} u_0 \geq -\frac{1}{6} \\ p^2 \geq 1 + 6u_0 \\ \Omega^2 = p^4 - (1 + 6u_0)p^2 \\ b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 + 3p^4} \end{cases} \tag{15}$$

Obviously, the $\cos p(x-t)$ is periodic, so the solution given by (14) is a periodic soliton solution with $x-t$-direction.(ref. fig.2)

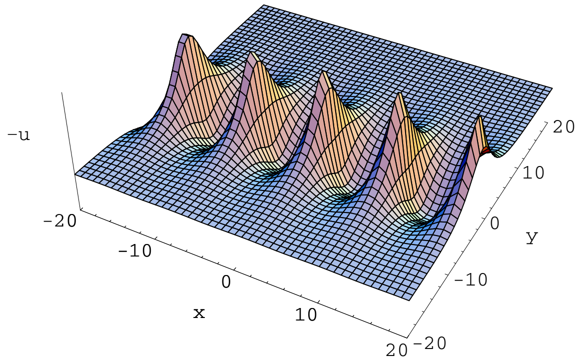


Figure 2: The $x-t$-periodic soliton solution for KPII equation as $u_0 = -\frac{1}{8}$

3.The doubly periodic solution to KPI and y-periodic soliton solution to KPII

Comparing (3) with (11), it is easily to find that the eq(3) may be changed into eq(11) and vice versa by using the temporal and spatial transformation $(\xi, y) \rightarrow (i\xi, iy)$. Because the solutions of KP equation are real functions, it is naturally to take specially $b_2 = 1$, $\gamma = 0$.And, making variable transformation $\xi \rightarrow i\xi$, $y \rightarrow iy$ in eq(6) and eq(12) yields

$$u = u_0 \pm \frac{2p^2[4b_1^2 + b_1(e^{p\xi} + e^{-p\xi})(e^{i\Omega y} + e^{-i\Omega y})]}{[b_1(e^{p\xi} + e^{-p\xi}) + (e^{i\Omega y} + e^{-i\Omega y})]^2} \tag{16}$$

Hence, the doubly periodic solution to KPI equation is obtained readily

$$u_3(x, y, t) = u_0 - \frac{2p^2[4b_1^2 + b_1(e^{ip(x-t)} + e^{-ip(x-t)})(e^{i\Omega y} + e^{-i\Omega y})]}{[b_1(e^{ip(x-t)} + e^{-ip(x-t)}) + (e^{i\Omega y} + e^{-i\Omega y})]^2} \tag{17}$$

i.e.

$$u_3(x, y, t) = u_0 - \frac{2p^2[b_1^2 + b_1 \cos(p(x-t)) \cos(\Omega y)]}{[b_1 \cos(p(x-t)) + \cos(\Omega y)]^2} \tag{18}$$

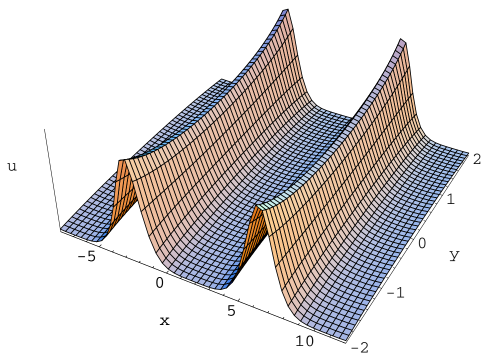
where

$$\begin{cases} u_0 \geq -\frac{1}{6} \\ p^2 \geq 1 + 6u_0 \\ \Omega^2 = p^4 - (1 + 6u_0)p^2 \\ b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 + 3p^4} \end{cases} \tag{19}$$

It is noted that the solution given by eq(18) is a singular periodic solution to KPI equation. In order to avoid the singularity, we set $\cos(p(x-t)) > 0$ and $\cos(\Omega y) > 0$.(ref.fig.3)

Besides, the y-periodic soliton solution to KPII is also given by

$$u_4(x, y, t) = u_0 + \frac{2p^2[4b_1^2 + b_1(e^{p(x-t)} + e^{-p(x-t)})(e^{i\Omega y} + e^{-i\Omega y})]}{[b_1(e^{p(x-t)} + e^{-p(x-t)}) + (e^{i\Omega y} + e^{-i\Omega y})]^2} \tag{20}$$



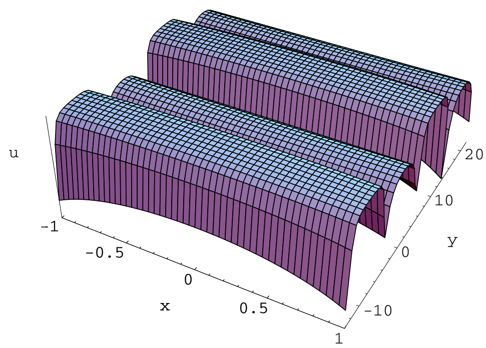


Figure 3: (a) (b)

(a) The doubly periodic solution for KPI equation with $x - t$ direction as $u_0 = -\frac{1}{8}$

(b) The doubly periodic solution for KPI equation with y direction as $u_0 = -\frac{1}{8}$

where

$$\begin{cases} u_0 < -\frac{1}{6} \\ p^2 < -\frac{1+6u_0}{4} \\ \Omega^2 = -p^4 - (1+6u_0)p^2 \\ b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 - 3p^4} \end{cases} \tag{21}$$

The solution given by (20) represents periodic soliton with y-direction.(ref.fig.4)

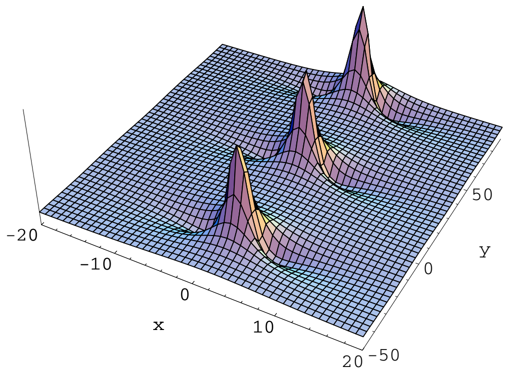
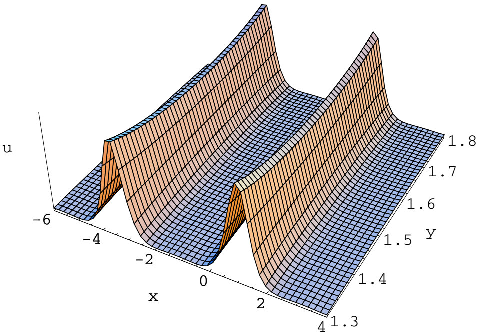


Figure 4: The y-periodic soliton solution for KPII equation as $u_0 = -\frac{1}{4}$

According to above discuss, we draw a conclusion that $u_0 = -\frac{1}{6}$ is a unique periodic bifurcation point for KPI and deflexion of soliton for KPII . Around the both sides at u_0, the property of solutions to KPI and KPII is all changed. As for KPI, when the equilibrium u_0 varies from one side of $-\frac{1}{6}$ to another side, two-soliton solution changes into doubly periodic solution. Whereas, the y-periodic soliton changes into x-periodic soliton for KPII. The two-soliton waves and doubly periodic soliton waves of KPI , periodic soliton waves on different spatial variable of KPII are interchanged around u_0.(ref.fig.5, fig.6)



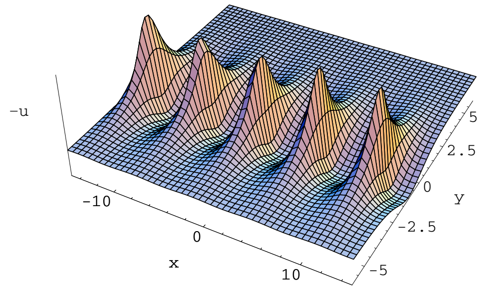


Figure 5: (a) (b)

(a) The doubly periodic solution for KPI equation as $u_0 = -\frac{1}{6}$

(b) The $x-t$-periodic soliton solution for KPII equation as $u_0 = -\frac{1}{6}$

Acknowledgements This work has been completed during first author visiting the Institute of Mathematical Sciences,The Chinese University of Hongkong (IMS,CUHK).This work is supported by The IMS,CUHK. Author like to thank professor Zhouping Xin for his valuable suggestions and help.

Reference

[1] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution and Inverse Scattering, Cambridge University Press, 1991.

[2] B. Konopechenko, Solitons in Multidimensions, Inverse Spectral Transform Method, World Scientific, 1993.

[3] B.B. Kadomtsev, V.I. Petviashvili,Stability of solitary waves in weakly dispersing mediums, Dokl. Akad. Nauk SSSR 192(1970)532.

[4] J. Satsuma,N-soliton of the two-dimensional Korteweg-de Vries equation, J. Phys. Soc. Jpn. 40(1976)286.

[5] S.V. Manakov, V.E. Zakharov, L.A. Bordag, A.R. Its, V.B.Matveev,Two-dimensional solitons of the Kadomtsov-Petviashvili equation and their interaction, Phys. Lett. A 63(1977)205.

[6] R.S. Johnson, S. Thompson,A soliton of the inverse scattering problem for the Kadomtsov-Petviashvili equation, Phys.Lett.A 66(1978)279.

[7] V.M. Galkin, D.E. Pelinovsky, Y.A. Stepanyants,The structure of the rational solutions to the Boussinesq equation, Physica D 80(1995)246.

[8] D.E. Pelinovsky,Bifurcation of new eigenvalues for the Benjamin-Ono equation, J. Math. Phys. 39(1998)5377.

[9] Q.P. Liu, M.Manas,Vevtorial Darboux transformation for the Kadomtsov-Petviashvili Hierarchy, J. Nonlinear Sci. 9(1999)213.

[10] J.W. Miles,On the Korteweg-de Vries equation for a gradually varying channel, J. Fluid Mech.79(1979)157.

[11] J.W. Miles,On surface-wave diffraction by a trench J. Fluid Mech.115(1982)315.

[12] Dai Z D,Jiang M R,Dai Q Y and Li S L,Homoclinic bifurcation for Boussinesq equation with even constraint,Chin.Phys.Lett. 23:5,2006,1065.