# DIFFRACTION OF A PLANAR RAREFACTION WAVE ALONG A COMPRESSIVE CORNER

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Abstract. The diffraction of a planar rarefaction wave along a compressive corner is considered by using numerical experiments and generalized characteristic analysis method. We have clarified that there are two patterns: regular reflection-like and Mach reflection-like (something like the diffraction of a planar shock along a compressive corner). And the reflection wave is always a compressive simple wave (called von Neumann wave), which goes into a critical transonic shock at last. The critical transonic shock is a special kind of transonic shock. The flow at the back bank of the shock is just sonic. A supersonic region near the corner appears in the second pattern.

## 1. Introduction

Diffraction of a planar shock along a compressive corner has long been an important problem in gas dynamics. It was considered initially by Mach (1878), who found two patterns: regular reflection and Mach reflection by experimental observation. von Neumann (1943) proposed a transition criterion between regular reflection and Mach reflection, using the method of shock polar. Bleakney and Taub (1949) delivered an analytical formulation of this criterion. The formulation is very complicated and completed by Chang & Chen (1986) and Sheng & Yin (to appear in ZAMP 2008) at last. Colella & Henderson (1990) and Zakhrian, Brio, Hunter & Webb (2000) found a new reflected wave: von Neumann wave in the diffraction of weak shock waves.

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Based on this problem, Zhang & Zheng (1990) proposed the two-dimensional Riemann problem for the compressible Euler equations. It is an initial value problem with initial data being constant in each quadrant and each jump in initial data outside of the origin projects one planar simple wave of shocks, rarefaction waves or contact discontinuities. Just as the diffraction of a planar shock along a compressive corner, the two-dimensional Riemann problem is self-similar. Their solutions are called pseudo-stationary flow. Using the generalized characteristic analysis, i.e., the analysis of characteristic, shock, sonic curve and the law of causality, they delivered a set of conjecture to the patterns of the solutions. The generalized characteristic analysis is a generalization of the classical characteristic analysis which was used to discuss many problems in Courant, R. & Friedrichs, K. O. (1948). Since then, numerical simulations for the two-dimensional Riemann problem have been performed by Schulz-Rinne, Collins & Glaz (1993), Chang, Chen & Yang (1995), Lax & Liu (1996, 1998), Li, Zhang & Yang (1998) and Kurganov & Tadmor (2002), among others. General patterns of both shock reflection and rarefaction wave reflection have been revealed. Li *et al.* (2006) proved that any wave adjacent to a constant state is a simple wave for the adiabatic Euler system in pseudo-stationary flow.

Recently, Glimm *et al.* (2008) present numerical evidence and generalized characteristic analysis to establish the existence of a transonic shock in such a 2-D Riemann problem, defined by the interaction of four rarefaction waves.

As a continuation of the last paper mentioned above, we are concerned with diffraction of a planar rarefaction wave along a compressive corner in the present paper.

We use the positive scheme (Lax & Liu 1996) which is a second order accurate for smooth solutions and first order accurate near shock waves. According to the numerical solutions, we draw pseudo characteristic curves, pseudo stream lines, pseudo sonic curves and shocks, and then do some analysis on their relationships. We discover there are two kinds of reflections in all including von Neumann waves and transonic shocks as follows:

1. Regular reflection-like (analogous to regular reflection in shock reflection).

The entire rarefaction wave strikes on and is reflected at the rigid wall. The reflected wave is a compressive simple wave (called von Neumann wave) which goes into a *critical transonic shock*.

2. Mach reflection-like (analogous to Mach reflection in shock reflection).

The entire rarefaction wave is divided into two parts. The former part ia the same as the above, but the tail of it is sonic. From then on, the rest part hits a sonic curve before they attach the rigid wall and is reflected at the sonic curve. The sonic curve plays the role of the triple point in Mach reflection of shock. The reflected wave is the same as the former one. Besides, a rarefactive supersonic region appears in the neighborhood of the corner. As the strength of the rarefaction wave is larger than a critical value, the supersonic region is connected with the von Neumann wave.

## 2. Set-up the problem for diffraction of a planar rarefaction wave along a compressive corner

### 2.1. **Generalized characteristic analysis.**

We consider the Euler equations

$$\begin{cases} \rho_t + \nabla \cdot (\rho U) = 0, \\ (\rho U)_t + \nabla \cdot (\rho U \otimes U) + \nabla p = 0, \\ (\rho E)_t + \nabla \cdot ((\rho E + p)U) = 0, \end{cases} \tag{2.1}$$

for the variables$(\rho, U, E)$, where $\rho$ is the density, $U = (u, v)$ is the velocity, $p$ is the pressure, $E = \frac{1}{2}|U|^2 + e$ is the specific total energy, and $e$ is the specific internal energy. For a polytropic gas, the pressure $p$ is defined by the equation

$$e = \frac{p}{(\gamma - 1)\rho},$$

where $\gamma > 1$ is a constant, and $\gamma = 1.4$ for air. In this paper, we take $\gamma = 1.4$.

Since system (2.1) is invariant under a dilation $(t, x, y) \to (kt, kx, ky)(k > 0)$, its solution must be self-similar, that is $(u, v, p, \rho)(t, x, y) = (u, v, p, \rho)(\xi, \eta)$, $(\xi, \eta) = (x/t, y/t)$, called pseudo-stationary flow.

At this moment, (2.1) reads

$$\begin{cases} -\xi\rho_\xi - \eta\rho_\eta + (\rho u)_\xi + (\rho v)_\eta = 0, \\ -\xi(\rho u)_\xi - \eta(\rho u)_\eta + (\rho u^2 + p)_\xi + (\rho uv)_\eta = 0, \\ -\xi(\rho v)_\xi - \eta(\rho v)_\eta + (\rho uv)_\xi + (\rho v^2 + p)_\eta = 0, \\ -\xi(\rho E)_\xi - \eta(\rho E)_\eta + (\rho u(E + \frac{p}{\rho}))_\xi + (\rho v(E + \frac{p}{\rho}))_\eta = 0. \end{cases} \tag{2.2}$$

Denote $(U, V) = (u - \xi, v - \eta)$, which is called pseudo-velocity. After simple calculation, systems (2.2) can, for smooth solutions, be reduced to

$$\begin{pmatrix} U & 0 & \rho & 0 \\ 0 & 1 & \rho U & 0 \\ 0 & 0 & 0 & \rho U \\ 0 & U & \gamma p & 0 \end{pmatrix} \begin{pmatrix} \rho_\xi \\ p_\xi \\ U_\xi \\ V_\xi \end{pmatrix} + \begin{pmatrix} V & 0 & 0 & \rho \\ 0 & 0 & \rho V & 0 \\ 0 & 1 & 0 & \rho V \\ 0 & V & \gamma p & 0 \end{pmatrix} \begin{pmatrix} \rho_\eta \\ p_\eta \\ U_\eta \\ V_\eta \end{pmatrix} + \begin{pmatrix} 2\rho \\ \rho U \\ \rho V \\ 2\gamma p \end{pmatrix} = 0 \tag{2.3}$$

The characteristic equation is

$$\rho^2(V - \lambda U)^2[(V - \lambda U)^2 - c^2(1 + \lambda^2)] = 0, \tag{2.4}$$

which gives either

$$\lambda_0 = \frac{V}{U} = \frac{v - \eta}{u - \xi} \qquad \text{(pseudo-stream characteristic)} \tag{2.5}$$

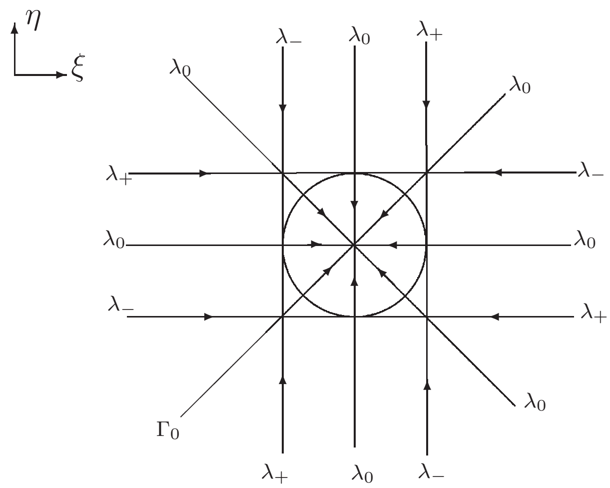


FIG.2.3.

or

$$\lambda_\pm = \frac{(u-\xi)(v-\eta) \pm c\sqrt{(u-\xi)^2+(v-\eta)^2-c^2}}{(u-\xi)^2-c^2} \quad \text{(pseudo-wave characteristic)} \tag{2.6}$$

where $c^2 = \gamma p/\rho$ is the sonic speed. The pseudo-Mach number

$$M = \sqrt{(u-\xi)^2+(v-\eta)^2}/c. \tag{2.7}$$

The flow is transonic and must be supersonic at infinity for bounded solutions.

For simplicity, "pseudo-" will be omitted in the following.

For the analysis of our numerical results in the following, we show the generalized characteristic analysis for constant state, rarefaction wave and shock, which were proved by Zhang & Zheng (1990).

(i) Constant states: $(\rho, u, v, p) = (\rho_0, v_0, u_0, p_0)$.

Their sonic curve is a circle:

$$(\xi - u_0)^2 + (\eta - v_0)^2 = c_0^2$$

The flow is subsonic inside the circle and supersonic outside the circle. The stream lines are all rays starting from infinity and focusing at the center of the sonic circle.

The wave characteristic lines are straight lines. They are perpendicular to the stream lines at the sonic circle and must be the tangent lines of the circle. They come from infinity and end at the sonic circle (see Fig.2.3).

(ii) $R_{12}^+$: connecting two constant states $i := (u_i, v_i, p_i, \rho_i), i = 1, 2$, (see Fig.2.4).

$$R_{12}^+(\xi): \begin{cases} \xi = u + \sqrt{p'(\rho)}, \quad (\xi_2 = u_2 + c_2 \le \xi \le \xi_1 = u_1 + c_1) \\ u = u_1 + \displaystyle\int_{\rho_1}^{\rho} \frac{\sqrt{p'\rho}}{\rho} d\rho, \quad (0 \le \rho_2 \le \rho \le \rho_1), \\ v = v_1, \quad \rho < \rho_1, \quad p_1\rho_1^{-\gamma} = p\rho^{-\gamma}. \end{cases} \tag{2.8}$$

The sonic curve is a straight segment

$$\eta = v_1 \quad (\xi_2 \le \xi \le \xi_1)$$

which we call the sonic stem.

The sonic stem is located on the stream line $\eta = v_1$ and is perpendicular to the $\lambda_\pm$ characteristic lines there. The other stream lines are symmetric about it and all intersect at point $(u_2, v_2)$. The sonic curve is made of the sonic circle of state ② and the sonic stem of $R_{12}^{+}(\xi)$. The sonic circle of state ① is located in $\xi \le \xi_1$. Therefore, it is an imaginary one.

(iii) $S_{12}^{\pm}$: connecting two constant states ① and ② (see Fig.2.5).

$$S_{12}^{\pm}(\xi): \begin{cases} \xi = \sigma_{12} = u_2 + \sqrt{\dfrac{\rho_1}{\rho_2} p'_{12}} = u_1 + \sqrt{\dfrac{\rho_2}{\rho_1} p'_{12}}, \\ v_2 = v_1, \\ \dfrac{p_2}{p_1} = \dfrac{(\gamma+1)\rho_2 - (\gamma-1)\rho_1}{(\gamma+1)\rho_1 - (\gamma-1)\rho_2}, \\ \rho_2 > \rho_1 \Leftrightarrow u_2 > u_1 \ (\rho_2 < \rho_1 \Leftrightarrow u_2 < u_1). \end{cases} \tag{2.9}$$

Obviously, $u_2 + c_2 > \sigma_{12} > u_1 + c_1, \sigma_+ > u_2$, which means that the sonic circle of state ① is located in $\xi < \sigma_{12}$, and the sonic circle of state ② is divided into parts by the shock. The characteristic lines of state ① and ② are determined by these two sonic circles, respectively.

*Remark* 2.1. We call $\overline{AB}$ transonic shock, the points $A$ and $B$ critical transonic shock and the rest of straight line AB supersonic shock, see Fig.2.5.

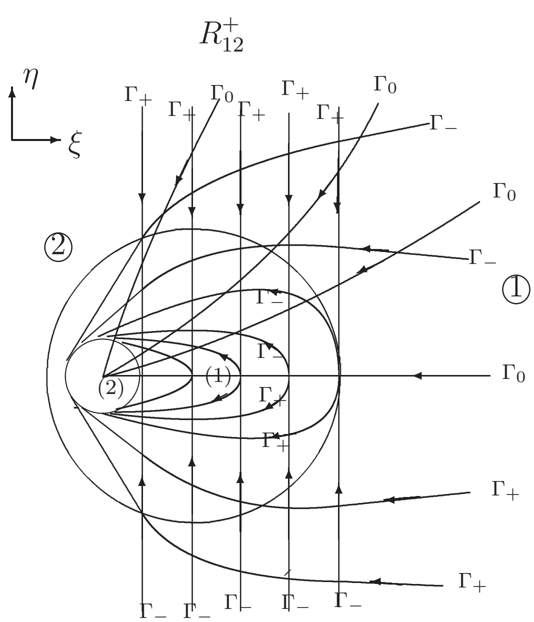


FIG.2.4.

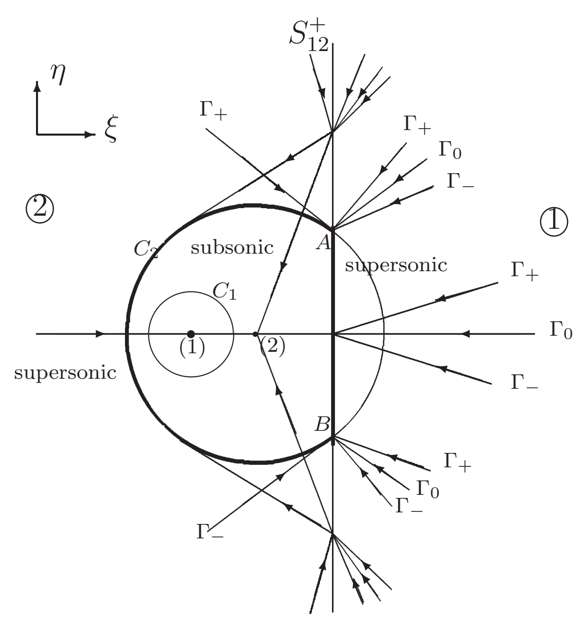


FIG.2.5.

2.2. **Set-up the problem.**

The problem for diffraction of a planar rarefaction wave along a compressive corner is an initial boundary value problem (2.1) with the initial data

$$(u,v,p,\rho)\Big|_{t=0} = \begin{cases} (u_1,v_1,p_1,\rho_1), & 0<x<+\infty, \quad x\tan\theta<y<+\infty \\ (u_2,v_2,p_2,\rho_2), & -\infty<x<0, \end{cases} \tag{2.10}$$

in region $\Omega$ with the boundary $\Gamma=\Gamma_1\cup\Gamma_2$, and the boundary condition

$$\begin{cases} (v-u\tan\theta)|_{\Gamma_1}=0, & x>0, t\geq 0 \\ v|_{\Gamma_2}=0, & x<0, t\geq 0. \end{cases} \tag{2.11}$$

where $\rho_2<\rho_1, u_2>u_1, \theta$ is the angle of compressive corner and ⓘ $=(u_i,v_i,p_i,\rho_i), i=1,2$ are constants. See Fig.2.6.

For the compatibility of initial and boundary conditions, we have

$$u_1=0, \quad v_1=u_1\tan\theta, \tag{2.12}$$

which implies

$$v_1=v_2=u_1=0. \tag{2.13}$$

The initial and boundary conditions (2.10) and (2.11) are invariant under the selfsimilar transformation. Therefore, we may seek the selfsimilar solutions for our problem. Moreover, the initial boundary value problem (2.1) with (2.10) and (2.11) are reduced to the boundary value problem (2.2) (see Fig.2.7) with

$$\begin{cases} (v-u\tan\theta)|_{\Gamma_1}=0 \\ v_2|_{\Gamma_2}=0. \end{cases} \tag{2.14}$$

$$\lim_{\xi^2+\eta^2\to+\infty}(u,v,p,\rho)(\xi,\eta) = \begin{cases} (u_1,v_1,p_1,\rho_1), & \xi>0,\ \eta>\tan\theta\,\xi, \\ (u_2,v_2,p_2,\rho_2), & \xi<0,\ \eta>0. \end{cases} \tag{2.15}$$

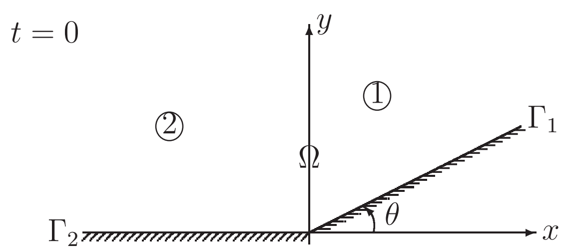


FIG.2.6.

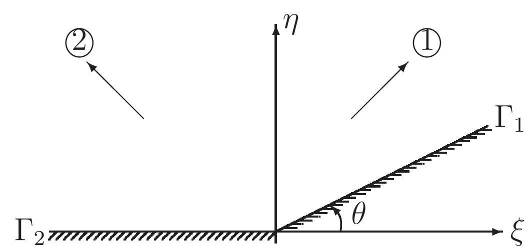


FIG.2.7.

From $\rho_2 < \rho_1, u_2 > 0$, we obtain a forward planar rarefaction wave denoted by $R_{12}^{+}$, connecting ① and ②, satisfying:

$$R_{12}^{+}: \begin{cases} p_1\rho_1^{-\gamma} = p_2\rho_2^{-\gamma}, \\ u_2 = u_1 + \dfrac{2\sqrt{\gamma}}{\gamma-1}\left(\sqrt{\dfrac{\rho_2}{\rho_1}} - \sqrt{\dfrac{p_1}{p_2}}\right), \\ v_1 = v_2, \quad \rho_2 < \rho_1. \end{cases} \tag{2.16}$$

## 3. A critical transonic shock formed by a compressive simple wave in regular reflection-like and Mach reflection-like

In this section, we do some numerical simulations to the problem and use generalized characteristic analysis to study the numerical results.

Analogous to [10], the computational domain is a rectangle $[-1.0, 0.75] \times [0, 0.75]$ and discrete time is $t = T_0$. Firstly, we discuss the algorithm for characteristics. The characteristic curves starting at the top boundary of rectangle belong to the family $\lambda_+$. Starting at the top boundary, we solve for $\lambda_+$ to obtain the pseudo characteristic curves, using a second order accurate Runge-Kutta scheme. The solution for $\lambda_+$ is continued to the sonic curve or the rigid wall. For the reflected characteristics $\lambda_-$ at the sonic curve or the rigid wall, we repeat the above processes.

In all numerical calculations, we fix the computational time $T_0 = 0.25$, the mesh grid $dx = dy = 1/800$, $\lambda^x = \lambda^y = 0.25$, the angle of the compressive corner $\theta = 45°$, using positive scheme (Lax & Liu 1996) which involves two limits $\alpha$ and $\beta$ set to be 0.9 and 0.1 and let the initial values be $p_1 = 1.0$, $\rho_1 = 1.0$, $u_1 = v_1 = 0$, $v_2 = 0$.

The numerical results show that there are two kinds of reflections to the problem.

### 3.1. **Regular reflection-like.**

Denote $p_1/p_2$ the strength of the planar rarefaction wave. In this case, we take $p_2 = 0.7045$. Our numerical results are shown in Fig.3.1.(a)-(e).

All the $\lambda_+$ characteristics of rarefaction wave $R_{12}^{+}$ coming from infinity attach the rigid wall $\overline{A_1A_2}$ and are reflected to form a simple wave (von Neumann wave) which is located between straight $\lambda_-$ characteristics $\overline{PB_1}$ and $\overline{A_2B_2}$, The $\lambda_-$ characteristics of the simple wave hit $\widehat{B_1B_2}$ of the sonic curve $\widehat{B_1D}$.

Some of the $\lambda_+$ characteristics of constat state ② coming from infinity penetrate the von Neumann wave and attach the rigid wall $\overline{A_2C}$, where the point $C$ is sonic. The neighboring $\lambda_+$ characteristics of constat state ② coming from infinity penetrate the von Neumann wave and hit the sonic curve $\widehat{CD}$ before attaching the rigid wall. Both the rest $\lambda_+$ characteristics and $\lambda_-$ characteristics, which are reflected at the rigid wall $\overline{A_1C}$ and the sonic curve $\widehat{CD}$, are

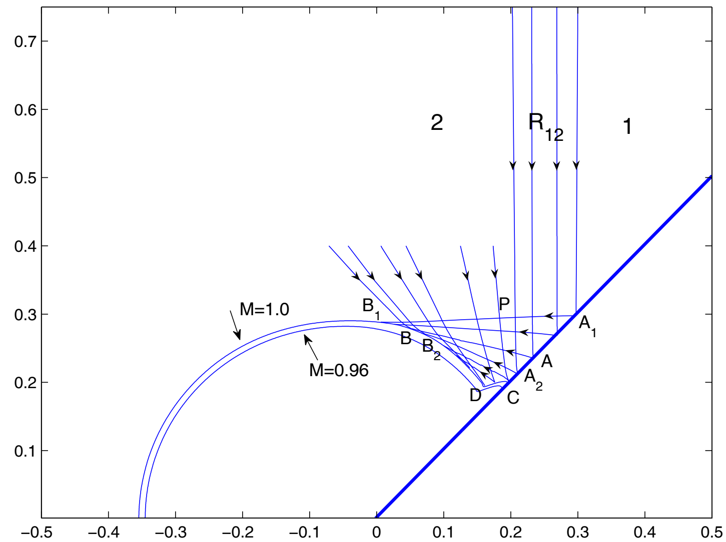


Fig.3.1.(a) Characteristic curves (light, arrow),
Mach number contours (light) with $M = 1.0, M = 0.96$ and rigid wall (bold).

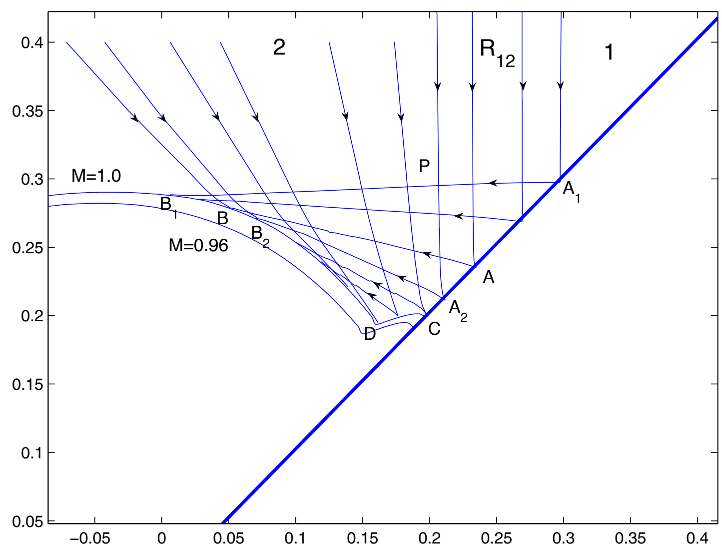


Fig.3.1.(b) Enlarged view from Fig.3.1.(a) shows the non parallel termination
of characteristics on the $M = 1.0$ contour and shock existence at the point B.

incoming to the sonic curve $\widehat{B_1D}$, not tangentally (Fig.3.1.(a)-(b)). The numerical results in Fig.3.1.(c) show that at a arbitrary point B of the sonic curve $\widehat{B_1D}$ , $\lambda_+(B) \neq \lambda_-(B)$, which means that B is supersonic. The contradiction indicates that $\widehat{B_1D}$ is a transonic shock. The transonic shock is different from the common transonic shock as at back bank of shock is sonic curve. This is a critical situation of transonic shock. We call it critical transonic shock. The theoretical foundation of conception of the critical transonic shock may refer to remark 2.1 and Fig.2.5 in Section 2.

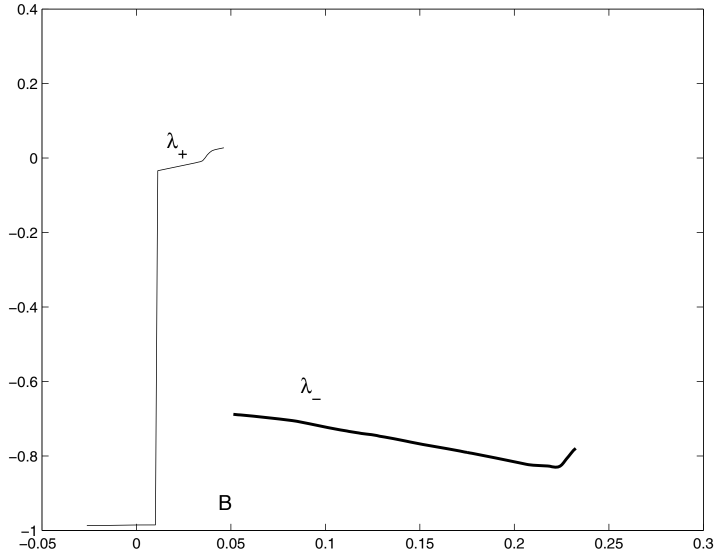


Fig.3.1.(c) The characteristics $\lambda_+$ from ② to B and $\lambda_-$ along AB meet at B, note that $\lambda_+ \neq \lambda_+$ at the point B.

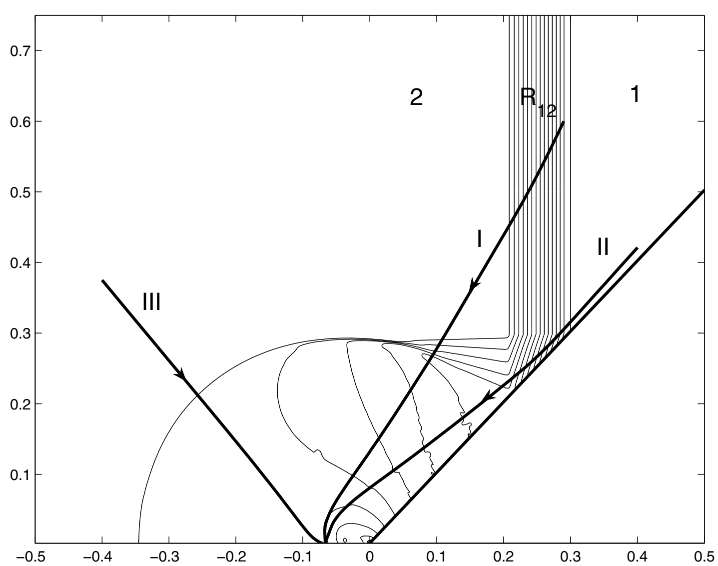


Fig.3.1.(d) Contour curves of density (light) and stream lines (bold, arrow).

There is another test for existence of a shock as follows. We notice the fact that

$$t\frac{\mathrm{d}\rho(t,x,y)}{\mathrm{d}t} = t(1,u,v)\cdot(\partial_t,\partial_x,\partial_y) = (u-\xi,v-\eta)\cdot(\partial_\xi,\partial_\eta) = \frac{\mathrm{d}\rho(\xi,\eta)}{\mathrm{d}s},$$

where $\frac{\mathrm{d}\rho(t,x,y)}{\mathrm{d}t}$ and $\frac{\mathrm{d}\rho(\xi,\eta)}{\mathrm{d}s}$ are the directional derivative of the density $\rho$ along the trajectories of gas particles in the $(t,x,y)$-space and along the pseudo-stream line, respectively([19], [10]). Therefore, $\frac{\mathrm{d}\rho}{\mathrm{d}s} > 0$ means a compression and the jump indicates a shock. The von Neumann wave is compressive and a critical transonic shock is formed, see Fig.3.1.(d)-(e).

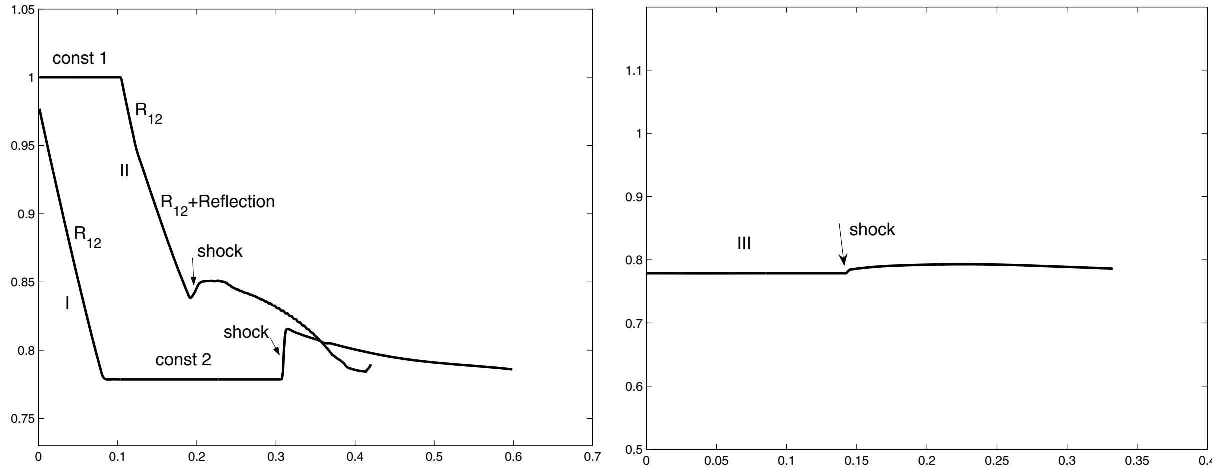


Fig.3.1.(e) Plot curves of density along stream lines I, II and III.

### 3.2. **Mach reflection-like.**

Let $p_2$ decrease, i.e., the strength of the planar rarefaction wave $R_{12}^{+}$ increase. Numerical results show that there exists a critical pressure $p_2 = p_c$, the $\lambda_+$ characteristic corresponding to $p_c$ attaches the rigid wall just at sonic point C. After that, the $\lambda_+$ characteristics hit the sonic curve $\widehat{CA_2}$ and are reflected as $\lambda_-$ characteristics, see Fig.3.2.(a). Similar to regular reflection-like, all the $\lambda_-$ characteristics go into critical transonic shock $\widehat{A_2B_1}$.

Besides, a rarefactive supersonic region appears in the neighborhood of the corner. The boundary of the supersonic region consists of sonic curve and transonic shock, see Fig.3.2.(a)-(d).

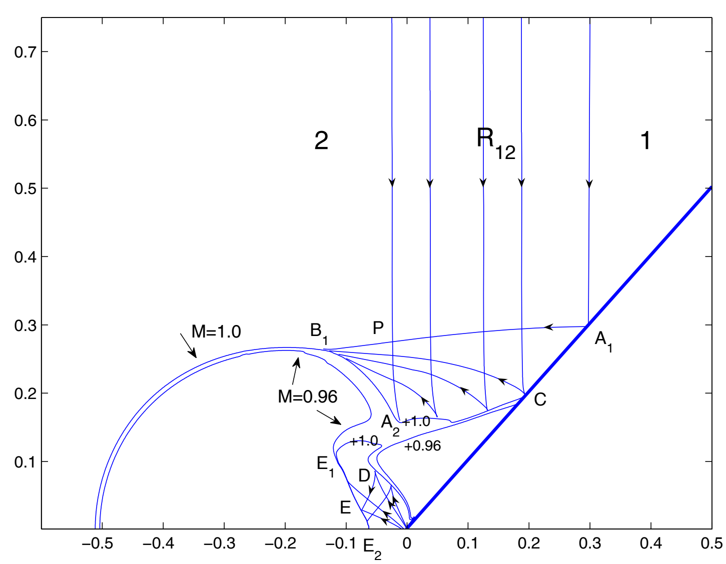


Fig.3.2.(a) Characteristic curves (light, arrow),
Mach number contours (light) with $M = 1.0, 0.96$ and rigid wall (bold).

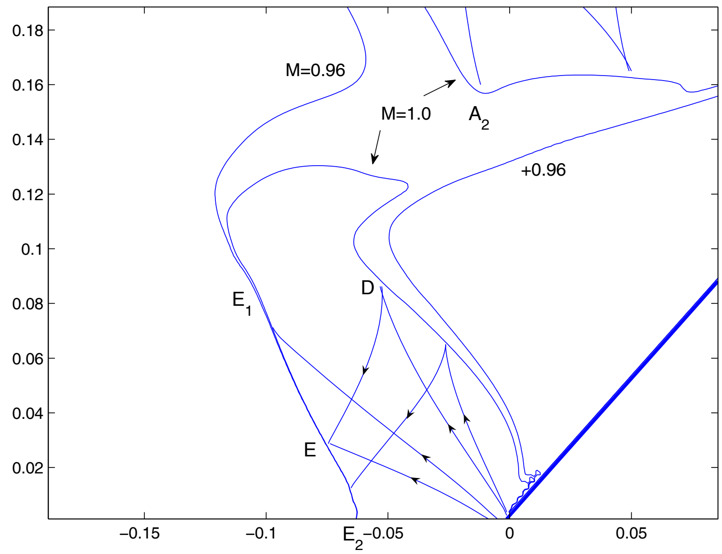


Fig.3.2.(b) Enlarged view from Fig.3.2.(a) shows the non parallel termination of characteristics on the $M = 0.96$ contour and shock existence at the point E.

Let us compare the diffraction of rarefaction wave with shock. In the regular reflection of shock, the incident shock is reflected at the rigid wall, and in the Mach reflection, the shock is reflected at the triple point separating from the rigid wall. While in regular reflection-like of the diffraction of rarefaction wave, the entire rarefaction wave is reflected at the rigid wall. In Mach reflection-like, part of the rarefaction wave is reflected at the rigid wall and the rest is reflected at the sonic curve separating from the rigid wall. That is why we call the two reflection regular reflection-like and Mach reflection-like of rarefaction wave.

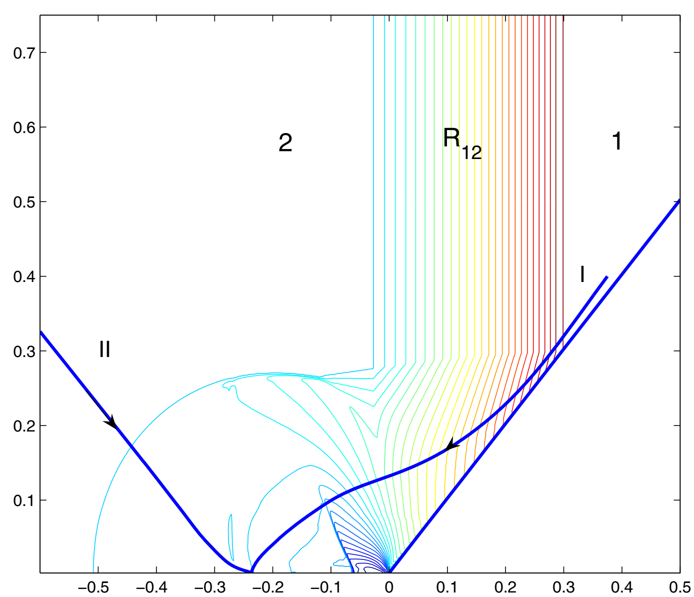


Fig.3.2.(c) Contour curves of density (light) and stream lines (bold, arrow).

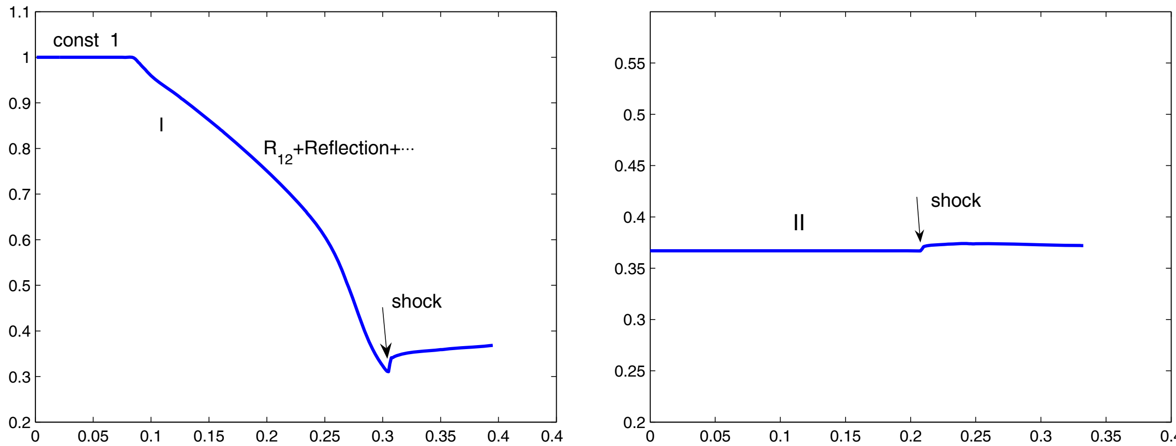


Fig.3.2.(d) Plot curves of density along stream lines I and II.

3.3. **The supersonic regions near and outside the corner join together.**

Numerical results show that as the strength of the rarefaction wave $R_{12}^{+}$ increases, the supersonic region becomes larger and larger. At last, it attaches point $A_2$ and the two supersonic regions join together. The critical pressure $p_2$ is denoted as $p_T$.

As an example of this case, we deliver the numerical results for the case $p_2 = 0.2058$. Analogous to Mach reflection-like, there exists a unique $\lambda_+$ characteristic, which attaches the rigid wall just at a sonic point C. The $\lambda_+$ characteristics located right to it attach the rigid wall $\overline{A_1C}$ and the rest ones hit the sonic curve $\widehat{CA_2}$, then all the $\lambda_+$ characteristics are reflected to form a compressive von Neumann wave and end up to a critical transonic shock. Besides, the supersonic region in the neighborhood of the corner and the upper supersonic region are joined together, see Fig.3.3.(a)-(c).

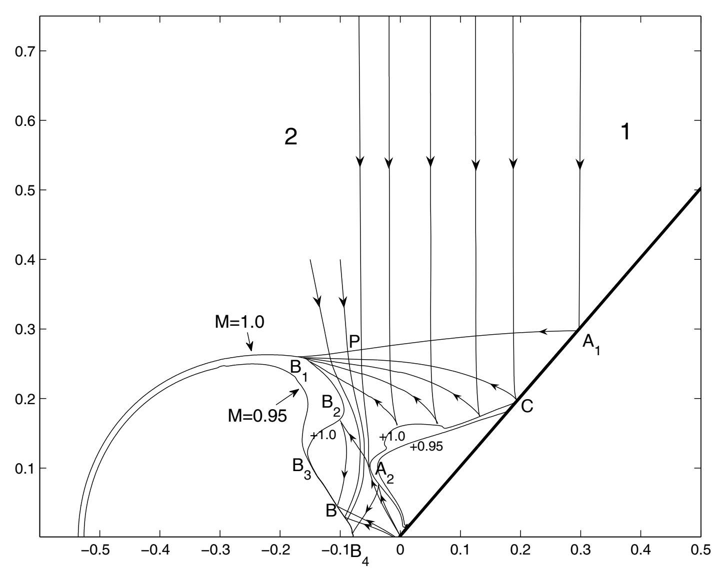


Fig.3.3.(a) Characteristic curves (light, arrow),
Mach number contours (light) with $M = 1.0, 0.96$ and rigid wall (bold).

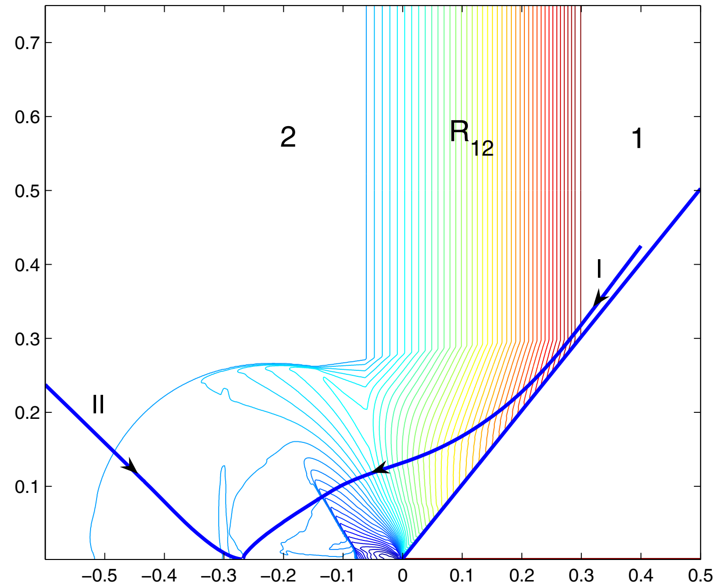


Fig.3.3.(b) Contour curves of density (light) and stream lines (bold, arrow).

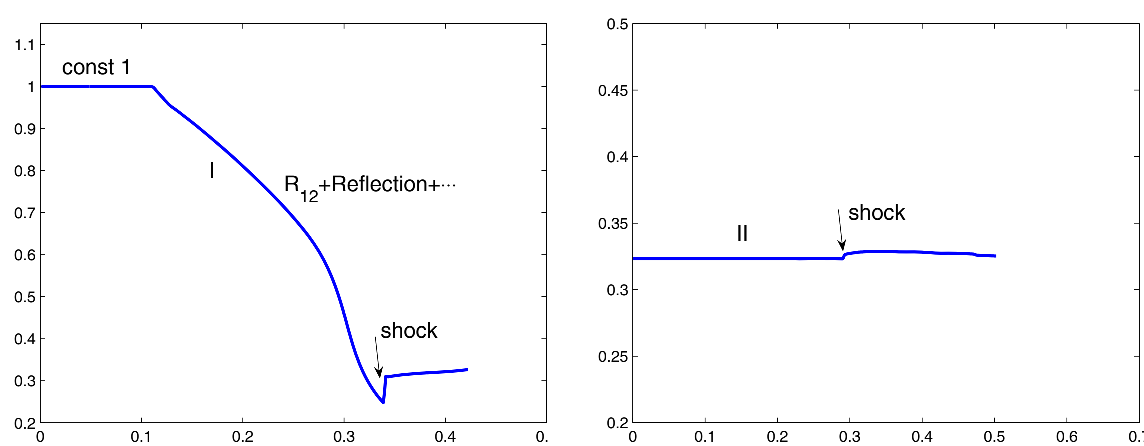


Fig.3.3.(c) Plot curves of density along stream lines I and II.

## 4. Conclusions

In this paper, we present numerical solutions of the Euler equations for the diffraction of a rarefaction wave along a compressive corner by use of positive scheme (Liu & Lax 1996, Lax & Liu 1998), which involves two limits $\alpha$ and $\beta$ set to be 0.9 and 0.1, the space mesh size $dx = dy = 1/800$ and CFL number $\lambda = 0.25$. And then we deliver generalized characteristic analysis according to numerical solutions. To demonstrate numerical results, we select the following angle of the compressive corner: 15°, 30°, 45°, 60°, 75°. Keeping among angles invariable, we increase the strength of the rarefaction wave $R_{12}^{+}$ and find that there are two kinds of reflections above mentioned, i.e., regular reflection-like, Mach reflection-like. The critical pressure value $p_c$ between the regular reflection-like and the Mach reflection-like are

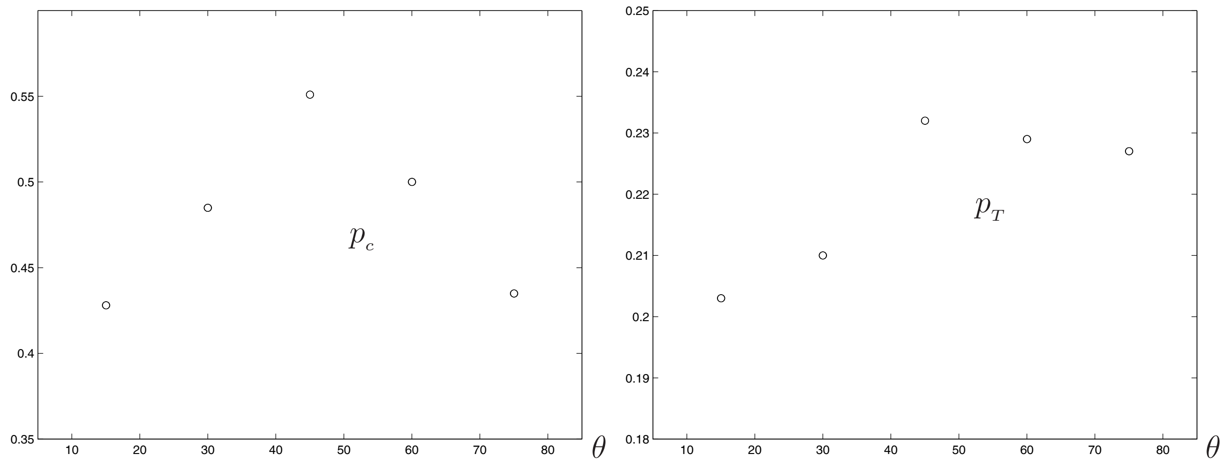


Fig.4.1. The critical pressures $p_c$ (left) and $p_T$ (right) at different angles $\theta$.

shown in Fig.4.1 (left) according to numerical calculations. From Fig.4.1 (left), we may see that $p_c$ is larger near $\theta = 45°$ than other angles. Numerical evidences show that the strength of the compressible simple reflected is the weaker near $\theta = 45°$ than any other angle keeping other parameters invariable. When the pressure $p_2$ is smaller than the $p_c$, there appears a supersonic region in the neighborhood of the corner. Furthermore, the critical angle $p_T$ is shown in Fig.4.1 (right). $p_T$ is larger near $\theta = 45°$ than other angles.

A critical transonic shock, at the back bank of which is just sonic is observed. There appears a von Neumann wave, which is a compressive simple wave and goes into the critical transonic shock at last, in diffraction of a planar rarefaction wave along a compressive corner.

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