ASIAN J. MATH.
Vol. 1, No. 1, pp. 1–16, March 1997

001

PARAMETRIZATION OF LOCAL BIHOLOMORPHISMS OF REAL ANALYTIC HYPERSURFACES*

M. S. BAOUENDI†, P. EBENFELT‡, AND LINDA PREISS ROTHSCHILD†

0. Introduction. A real analytic hypersurface $M \subset \mathbb{C}^N$ is said to be *finitely nondegenerate* at $p_0 \in M$ if there is a positive integer k such that

$$\text{span } \{L^\alpha \rho_Z(p_0, \bar{p}_0) \colon |\alpha| \le k\} = \mathbb{C}^N. \tag{0.1}$$

Here $L_1, ..., L_{N-1}$ is a basis for the CR vector fields on M near p_0, L^α denotes $L_1^{\alpha_1} ... L_{N-1}^{\alpha_{N-1}}$, $\rho(Z, \bar{Z}) = 0$ is a defining equation of M near p_0, and $\rho_Z = \partial\rho/\partial Z$. M is called *k-nondegenerate* at p_0 if k is the smallest integer for which (0.1) holds. This integer is an invariant of M at p_0; it does not depend on the basis $L_1, ..., L_{N-1}$, the defining function $\rho(Z, \bar{Z})$, or the coordinates used (see [BHR] in which this notion was introduced). The hypersurface M is 1-nondegenerate at p_0 if and only if it is Levi nondegenerate at p_0.

A connected hypersurface M is called *holomorphically nondegenerate* if there is no nontrivial germ of a holomorphic vector field (i.e. a holomorphic section of $T^{(1,0)}(\mathbb{C}^N)$) tangent to M. One can show that it suffices to check this condition at any one point of M. Furthermore, a connected real analytic hypersurface M is holomorphically nondegenerate if and only if it is finitely nondegenerate at some point (and hence at all points outside a proper real analytic subset of M). We refer the reader to [BHR], [BER1] for details. (See also Stanton [S2].)

In this paper we study germs of biholomorphisms mapping one real analytic hypersurface into another and show that at points of finite nondegeneracy there is a natural parameter space which has the structure of a real analytic manifold.

Some of the main results of this paper can be described as follows. Let M and M' be two real analytic hypersurfaces k_0-nondegenerate at p and p' respectively. We consider the (possibly empty) set of germs H at p of invertible holomorphic mappings in $\mathbb{C}^N$ with $H(M) \subset M'$ and $H(p) = p'$. This is a subset of the space of all germs at p of holomorphic mappings $\mathbb{C}^N \to \mathbb{C}^N$ and as such inherits a natural inductive limit topology corresponding to uniform convergence on compact neighborhoods of p. We show (Theorem 1) that this set, equipped with its natural topology, is mapped into the complex jet group of order $2k_0$ of $\mathbb{C}^N$ homeomorphically onto a real analytic, totally real submanifold $\Sigma_{p,p'}$. Moreover, $\Sigma_{p,p'}$ is given by equations that can be explicitly constructed from defining equations of M and M'. In particular, the germs of hypersurfaces (M, p) and (M', p') are biholomorphically equivalent if and only if these equations have a solution; we say that (M, p) and (M', p') are biholomorphically equivalent if there is a germ H of a biholomorphism mapping M into M' with $H(p) = p'$. We say that (M, p) and (M', p') are *formally equivalent* if the components of H are assumed only to be formal power series at p. We show (Theorem 5) that if (M, p) and (M', p') are finitely nondegenerate, then they are formally equivalent if and only if

* Received January 1, 1997; accepted for publication (in revised form) February 6, 1997.

† Department of Mathematics, University of California at San Diego, La Jolla, CA 92093, USA (sbaouendi@ucsd.edu, lrothschild@ucsd.edu). The first and the third authors are partially supported by National Science Foundation grant DMS 95-01516.

‡ Department of Mathematics, Royal Institute of Technology, 100 44 Stockholm, Sweden (ebenfelt@math.kth.se). This author is supported by a grant from the Swedish Natural Science Research Council.