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POINTWISE DECAY TO CONTACT DISCONTINUITIES FOR SYSTEMS OF VISCOUS CONSERVATION LAWS*

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Abstract. In this paper we study the large time asymptotic behavior toward contact waves for a class of systems of viscous nonlinear conservation laws. Although a given contact discontinuity can not be an asymptotic attractor for the viscous system in general, yet for a large class of systems with some structure constraints, viscous contact waves, which are the exact solutions to the viscous systems and approximate the corresponding contact discontinuities on any finite interval, can be constructed explicitly and are shown to be asymptotically stable with small generic initial perturbations for the viscous systems provided that the strength of the contact discontinuities is suitably small. The leading order large time asymptotic form of the viscous solutions toward a contact wave can be determined a priorily by the distribution of the initial excessive mass, and is a superposition of the viscous contact wave (properly shifted) in the principal linearly degenerate field and nonlinear (or linear) diffusive waves in the transversal fields. The high order deviation of the viscous solutions from its leading order asymptotic ansatz is estimated pointwisely by constructing an approximate fundamental solution of the viscous system linearized around the viscous contact wave, and by analyzing the interactions among viscous contact waves and diffusive waves. The theory is applied to equations for one-dimensional polytropic gases.

1. Introduction. We consider the large time asymptotic behavior of the solutions to the initial value problem for the system of viscous conservation laws of the form

$$\partial_t u + \partial_x f(u) = \varepsilon \partial_x^2 u, \qquad x \in \mathbb{R}^1, \quad t > 0, \tag{1.1}$$

where $u = u(x,t)$, a vector-valued function in $\mathbb{R}^n$, represents density variables with $f(u)$ being its corresponding fluxes which is assumed to be smooth, and $\varepsilon > 0$ is a viscosity coefficient. (1.1) is supplemented with the initial data

$$u(x,0) = u_0(x) \tag{1.2}$$

satisfying

$$\lim_{x\to\pm\infty} u_0(x) = u_\pm, \tag{1.3}$$

where u_+ and u_- are two constant states in $\mathbb{R}^n$.

The qualitative behavior of the solutions to (1.1)–(1.3) is closely related to that of the corresponding inviscid system:

$$\partial_t u + \partial_x f(u) = 0. \tag{1.4}$$

It is assumed that (1.4) is hyperbolic, i.e., the Jacobian matrix $f'(u)$ has real eigenvalues $\lambda_1(u) \le \lambda_2(u) \le \ldots \le \lambda_n(u)$ with corresponding left and right eigenvectors $l_i(u)$ and $r_i(u)$ such that

$$\begin{aligned} L(u)f'(u)R(u) &= \Lambda \equiv \mathrm{diag}\{\lambda_1(u), \ldots, \lambda_n(u)\} \\ L(u)R(u) &= I \equiv \text{Identity matrix} \end{aligned} \tag{1.5}$$

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