ASIAN J. MATH.
Vol. 1, No. 1, pp. 96–180, March 1997

005

ZERO-LOOP OPEN STRINGS IN THE COTANGENT BUNDLE AND MORSE HOMOTOPY*

KENJI FUKAYA† AND YONG-GEUN OH‡

0. Introduction. Many important works in *symplectic geometry and topology* are regarded as *the symplectization* or *the quantization* of the corresponding results in *ordinary geometry and topology.* One outstanding example is the celebrated Arnold conjecture which concerns the number of fixed points of a symplectic diffeomorphism or that of intersection points of two Lagrangian submanifolds. The homological version of the conjecture has been proved in various cases (see [Fl1-5], [O2,3,6], [On] and [PSS], and [O7] for a survey and references on the Arnold conjecture and Floer homology). The estimate (in its homological version) predicted by the Arnold conjecture can be regarded as the symplectization or the quantization of the Morse inequality, and conversely the latter can be considered as the *semi-classical limit* and so a consequence of the former. From now on, we will use the term "quantization" for the similar process that appear below.

To illustrate this statement, we consider the cotangent bundle of a given compact manifold and the graphs of exact one forms. The graph of an exact one form becomes a Lagrangian submanifold of the cotangent bundle with respect to the canonical symplectic structure. Then Floer's result on the Lagrangian intersections [Fl1,3] will imply the Morse inequality. The Lagrangian intersection theory is indeed the intersection theoretic version of the Morse theory, while the Lefsechtz intersection theory is that of the degree theory of generic vector fields.

The principle that the symplectic topology and geometry of the cotangent bundle (or more generally that of symplectic manifolds) is the quantization of the ordinary topology and geometry of the base, is a general principle which can be applied to many other situations. The equivalence of the two often holds, when there occurs the absence of the *quantum contribution* (or the non-existence of the *bubbling phenomena*). In this paper, we will provide another example of this principle in which we prove that the rational homotopy type of a compact manifold M can be described by the moduli space of pseudo holomorphic disks with appropriate Lagrangian boundary conditions in its cotangent bundle T^*M. The precise statement of our result is in Section 1.

Our result paves the way to applying the A^∞-structure introduced by the first author [Fu2] to the study of the estimate, in terms of the rational homotopy invariant of the base manifold, of the number of intersections of the zero section in the cotangent bundle and its Hamiltonian deformation. This enables us to go one step further, beyond the existing homological estimate in the literature, towards the proof of the original Arnold conjecture which states that the number of the intersections will be greater than or equal to the Morse number of M. Viterbo [V] and Eliashberg-Gromov [EG] have also studied this kind of estimate using the *generating functions* of Lagrangian submanifolds.

* Received January 31, 1997; accepted for publication February 21, 1997.

† Department of Mathematics, Faculty of Sciencess, Kyoto University, Kyoto, Kitashirakawa, JAPAN (fukaya@kusm.kyoto-u.ac.jp). Partially supported by Grant in-Aid for Scientific Research on Priority Areas 231 Infinite Analysis, Grant in-Aid for Scientific Research 07304010 and 07640111 and by Sumitomo foundation.

‡ Department of Mathematics, University of Wisconsin, Madison, WI 53706, USA (oh@math.wisc.edu). Partially supported by the NSF Grant DMS-9504455.