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CONFORMAL MODULES*

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Abstract. In this paper we study a class of modules over infinite-dimensional Lie (super)algebras, which we call conformal modules. In particular we classify and construct explicitly all irreducible finite conformal modules over the Virasoro and the N=1 Neveu-Schwarz algebras, over the (super)current algebras and their semidirect sums.

0. Introduction. Conformal module is a basic tool for the construction of free field realizations of infinite-dimensional Lie (super)algebras in conformal field theory. This is one of the reasons to classify and construct such modules. In the present paper we solve this problem under the irreducibility assumption for the Virasoro and the Neveu-Schwarz algebras, for the (super)current algebras and their semidirect sums. Since complete reducibility does not hold for conformal modules, one has to discuss the extension problem. This problem is solved in [1].

The basic idea of our approach is to use three (more or less) equivalent languages. The first is the language of local formal distributions, the second is the language of modules over conformal algebras, and the third is the language of conformal modules over the extended annihilation subalgebras. The problem is solved using the third language by means of the crucial Lemma 3.1. Note that conformal modules over Lie algebras of Cartan type were studied in [7], where, in particular, a proof of Corollary 3.3 is contained.

This paper is organized as follows. In Section 1 the concepts of a Lie (super)algebra of formal distributions and of a conformal (super)algebra are recalled. They are more or less equivalent notions. In Section 2 we introduce conformal modules over a Lie (super)algebra of formal distributions and clarify their connections with modules over the corresponding conformal (super)algebra. We then show that modules over a conformal (super)algebra are in 1-1 correspondence with modules over the extended annihilation subalgebra of the associated Lie (super)algebra of formal distributions. At the end of Section 2 examples of conformal modules over the Virasoro, the (super)current and Neveu-Schwarz algebras and their various semidirect sums are constructed. In Section 3 we first prove the key lemma (Lemma 3.1) and with its help classify all irreducible finite conformal modules over the (extended) annihilation subalgebra of the above-mentioned Lie (super)algebras. The main result, stated in Theorem 3.2, which describes all finite irreducible modules over the conformal (super)algebras in question (hence all irreducible finite conformal modules over the corresponding Lie (super)algebras), is then immediate.

Roughly speaking, the main claim of the present paper is that all non-trivial modules over current, Virasoro, N=1 superconformal algebras and their semidirect sums are non-degenerate. For N>1 superconformal algebras interesting degeneracies occur in some non-trivial conformal modules. These effects are studied in [6].

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