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PRINCIPAL BUNDLES ON ELLIPTIC FIBRATIONS *

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Abstract. A central role in recent investigations of the duality of F-theory and heterotic strings is played by the moduli of principal bundles, with various structure groups G, over an elliptically fibered Calabi-Yau manifold (on the heterotic side), and specifically over an elliptic-K3-fibered Calabi-Yau, on the F-theory side. In this note we propose a simple algebro-geometric technique for studying the moduli spaces of principal G-bundles on an arbitrary variety X which is elliptically fibered over a base S: The moduli space itself is naturally fibered over a weighted projective base parametrizing spectral covers $\tilde{S}$ of S, and the fibers are identified as translates of distinguished Pryms of these covers. In nice situations, the generic Prym fiber is isogenous to the product of a finite group and an abelian subvariety of $Pic(\tilde{S})$.

1. Introduction. Moduli spaces of principal G-bundles on K3 surfaces, and on Calabi-Yaus of other dimensions, are basic ingredients of the compactification of heterotic strings, especially for structure groups $E8 \times E8$ and $SO(32)$. More recently, compactifications of F-theory [1] have suggested the special importance of moduli spaces of G-bundles on elliptically fibered Calabi-Yaus, with G a semisimple subgroup of $E8 \times E8$. The purpose of this note is to propose a general technique for studying G-bundles on (not necessarily CY) elliptic fibrations $X \to S$, with arbitrary semisimple structure group G.

The idea is quite simple. To a principal G-bundle P over X we associate its *spectral data.* This consists of the *cameral cover* $\tilde{S} \to S$, a Galois cover with covering group the Weyl group W of G; of a collection of multisection maps $v_\lambda : \tilde{S} \to X$, one for each character $\lambda \in \Lambda$ of the maximal torus $T \subset G$, subject to the condition of W-equivariance (the *spectral covers* are the various images of $\tilde{S}$ in X); and of a T-bundle over $\tilde{S}$, subject to a certain twisted form of W-equivariance. When the cameral cover is reasonably nice, it determines the *distinguished Prym* $\mathrm{Prym}(\tilde{S}/S)$, an extension of an abelian variety by a finite group. The G-bundles on X with a given cameral cover $\tilde{S}$ (and any consistent collection of maps to X) can then be parametrized by the distinguished Prym. An open subset of the moduli of G-bundles can thus be fibered over the parameter space for nice cameral covers, the fibers being the distinguished Pryms. When the cameral cover is not-so-nice, the description becomes less precise; but the whole construction has a local character, so bad behavior can be traced to specific singular loci. We avoid much of the difficulty by considering *regularized bundles*, or bundles with some additional structure, specified by a reduction of the structure group, over each $s \in S$, to an abelian subgroup which is the centralizer in G of a regular element. (We call such a subgroup a regular centralizer.) Nice bundles have a unique regularization, special ones may have large families of regularizations, and others will have none, but can become regularized after some blowing up in the base S.

The reason that regular centralizers are the right structure can be seen already from the behavior of G-bundles on a single elliptic curve E, and already for $G = SL(2)$. The structure group of "most" semistable G-bundles on E can be reduced to a maximal

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