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THE 14-DIMENSIONAL KERVAIRE INVARIANT AND THE SPORADIC GROUP M_{12} *

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There is a 14 dimensional aspect to the sporadic group M_{12}. For one thing

$$Syl_2(M_{12}) = Syl_2(G_2(q))$$

for $q \equiv 3, 5 \bmod (8)$ and the exceptional Lie group G_2 is 14 dimensional. Secondly, in [AMM1] we showed that the Poincaré series for $H^*(M_{12}; \mathbb{F}_2)$ is

$$\frac{1 + t^2 + 3t^3 + t^4 + 3t^5 + 4t^6 + 2t^7 + 4t^8 + 3t^9 + t^{10} + 3t^{11} + t^{12} + t^{14}}{(1-t^4)(1-t^6)(1-t^7)}$$

which suggests, since $\frac{1}{(1-t^4)(1-t^6)(1-t^7)}$ is the Poincaré series for the classifying space B_{G_2}, that there should be a fibering of the form

$$F \longrightarrow B^+_{M_{12}} \longrightarrow B_{G_2}$$

with the fiber a 14 dimensional Poincaré duality space. Indeed, in [M], (see also [BW]), I constructed just such a fibration.

There is one more way in which one might expect the sporadic nature of M_{12} to manifest itself. According to [R], the natural homotopy class in

$$\pi^s_{14}(S^0) = 2^2 = \langle \bar{\kappa}, \sigma^2 \rangle$$

associated to the reframing of G_2 is $\bar{\kappa}$, so one would expect that there was some way in which a homotopy class in $\pi^s_{14}(S^0)$ could be associated to F, and this class should be σ^2 or $\sigma^2 + \bar{\kappa}$. In other words, this class should represent an element in $\pi^s_{14}(S^0)$ of Kervaire invariant one. Indeed, it is the object of this note to show just that.

One connection between homotopy theory and group theory lies in the well known equivalence $\mathbb{Z} \times B^+_{\mathcal{S}_\infty} \simeq Q(S^0) = \lim_{n \mapsto \infty}(\Omega^n \Sigma^n)$, and there is a natural embedding $h: M_{12} \subset \mathcal{S}_{12}$, embedding M_{12} as a maximal subgroup of $\mathcal{S}_{12}$, that gives rise to a composition map

$$B_h: B^+_{M_{12}} \longrightarrow B^+_{\mathcal{S}_{12}} \hookrightarrow B^+_{\mathcal{S}_\infty} \simeq Q(S^0)$$

which, at the homotopy level induces a map $B_{h,*}: \pi_*(B^+_{M_{12}}) \longrightarrow \pi^s_*(S^0)$. Then the main result of this note is

THEOREM. *There is an element $A \in \pi^s_{14}(B_{M_{12}})$ with Hurewicz image $[F]$, the orientation class of the fiber F above, and every such A has image $B_h(A) = \sigma_7^2$ or $\sigma_7^2 + \bar{\kappa}$ under the composition above.*

The way in which the class A is constructed may have independent interest. The construction is based on the use of the stable transfer,

$$tr: Q(B_{G_2(q)}) \longrightarrow Q(B_{Syl_2(M_{12})}),$$

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