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K–THEORY FOR TRIANGULATED CATEGORIES I(A): HOMOLOGICAL FUNCTORS *

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0. Introduction. We should perhaps begin by reminding the reader briefly of Quillen's Q–construction on exact categories.

DEFINITION 0.1. *Let $\mathcal{E}$ be an exact category. The category $Q(\mathcal{E})$ is defined as follows.*

0.1.1. *The objects of $Q(\mathcal{E})$ are the objects of $\mathcal{E}$.*

0.1.2. *The morphisms $X \bullet\!\!\to X'$ in $Q(\mathcal{E})$ between $X, X' \in Ob(Q(\mathcal{E})) = Ob(\mathcal{E})$ are isomorphism classes of diagrams of morphisms in $\mathcal{E}$*

$$\begin{array}{ccc} X & & X' \\ & \searrow \quad \swarrow & \\ & Y & \end{array}$$

where the morphism $X \longrightarrow Y$ is an admissible mono, while $X' \longrightarrow Y$ is an admissible epi. Perhaps a more classical way to say this is that X is a subquotient of X'.

0.1.3. *Composition is defined by composing subquotients;*

$$\begin{array}{ccc} X & & X' \\ & \searrow \quad \swarrow & \\ & Y & \end{array} \qquad \text{and} \qquad \begin{array}{ccc} X' & & X'' \\ & \searrow \quad \swarrow & \\ & Y' & \end{array}$$

compose to give

$$\begin{array}{ccccccc} X & & & X' & & & X'' \\ & \searrow & \swarrow & & \searrow & \swarrow & \\ & Y & & PO & & Y' & \\ & & \searrow & & \swarrow & & \\ & & & Z & & & \end{array}$$

where the square marked PO is a pushout square.

The category $Q(\mathcal{E})$ can be realised to give a space, which we freely confuse with the category. The Quillen K–theory of the exact category $\mathcal{E}$ was defined, in [9], to be the homotopy of the loop space of $Q(\mathcal{E})$. That is,

$$K_i(\mathcal{E}) = \Pi_{i+1}\big[Q(\mathcal{E})\big].$$

Quillen proved many nice functoriality properties for his K–theory, and the one most relevant to this article is the resolution theorem. The resolution theorem asserts the following.

THEOREM 0.2. *Let $F : \mathcal{E} \longrightarrow \mathcal{F}$ be a fully faithful, exact inclusion of exact categories. Suppose further that every object $y \in \mathcal{F}$ admits a resolution*

$$0 \longrightarrow x_n \longrightarrow x_{n-1} \longrightarrow \cdots \longrightarrow x_1 \longrightarrow x_0 \longrightarrow y \longrightarrow 0,$$

with all the x_i's in $\mathcal{E}$. Then the natural map

$$Q(\mathcal{E}) \longrightarrow Q(\mathcal{F})$$

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