ASIAN J. MATH.
Vol. 1, No. 3, pp. 422-434, September 1997

002

A MIXED PRODUCT KRYLOV SUBSPACE METHOD FOR SOLVING NONSYMMETRIC LINEAR SYSTEMS*

TONY F. CHAN† AND QIANG YE‡

Abstract. In this paper, a product Krylov subspace method that we call mixed BiCGSTAB-CGS is derived. The method is built on the idea of the standard CGS and BiCGSTAB iterations but allows switching between the two at each iteration. This flexibility can be used, for example, to address the difficulty of excessive increase in residual norm in CGS, which may cause instability. In particular, a CGS based implementation will be presented, which can be regarded as another way of using the BiCGSTAB idea to improve the stability of CGS. Numerical examples are given to demonstrate the stabilizing effect of the mixed algorithm.

1. Introduction. Iterative methods for solving a large nonsymmetric linear systems $Ax = b$ that extract approximate solutions from the Krylov subspace $K_n = \text{span}\{b, Ab, A^2b, \cdots, A^nb\}$ are usually called Krylov subspace methods. The BiCG algorithm [4, 9] is a classical Krylov subspace method that produces an approximation x_n with a residual reduction $r_n = b - Ax_n = P_n(A)r_0$ ($r_0 = b$) where P_n is a polynomial of degree n and called the BiCG polynomial.

In the past few years, several efficient product Krylov subspace methods such as CGS [13] and BiCGSTAB [15] have been developed to accelerate convergence in BiCG and to avoid multiplications by the transpose of A (see [1, 8, 10] for a general review and comparison). These product type methods are based on constructing the product of the BiCG polynomial and some other accelerating polynomials. In comparison with BiCG, which requires $2n$ matrix-vector multiplications at iteration n, CGS uses the same amount of matrix-vector multiplications to produce a reduction with P_n^2, i.e., $r_n = P_n^2(A)r_0$. This potentially accelerates the convergence by a factor of 2. Unfortunately, during the early stage of iteration, $P_n(A)$ may not be a contraction and thus r_n may grow substantially or vary significantly in magnitude, causing instability or stagnation of the true residual [12]. A remedy to this problem is BiCGSTAB [15] (or BiCGSTAB(l) [7, 11]), which constructs $r_n = P_n(A)Q_n(A)r_0$ with Q_n being a product of polynomials of degree 1 (or l resp.) constructed locally to minimize the residual. Indeed, the BiCGSTAB type methods have proved to be very efficient in smoothing and accelerating the convergence and is currently one of the most popular methods. However, there are many cases where CGS remains competitive. For example, BiCGSTAB may suffer from a type of near-breakdown that is associated with BiCGSTAB only and may result in stagnation of residuals.

Since the difficulties encountered in CGS and BiCGSTAB are of different type and usually occur only at a small subset of the iteration steps, it might be advantageous to consider a combination of the two that can choose either of the two kinds of construction at each iteration and avoid using the one for which difficulties arise. Also local steepest descent is used throughout BiCGSTAB iterations, which may not be optimal in exploiting the global information available in the Krylov sequences. Therefore, it might still be desirable to construct residual reduction like $P_n^2(A)r_0$ but

*Received September 5, 1997; accepted for publication (in revised form) October 25, 1997.

† Department of Mathematics, UCLA, Los Angeles, CA 90095-1555, USA (chan@math.ucla.edu). Research supported by NSF DMS-9626755, NSF ASC-9720257, and ARO contract DAAL-03-91-C-0047 (Univ. Tenn. subcontract ORA4466.04 Amendment 1 and 2).

‡Department of Applied Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2 (ye@newton.amath.umanitoba.ca). Research supported by a grant from Natural Sciences and Engineering Research Council of Canada.