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K–THEORY FOR TRIANGULATED CATEGORIES I(B): HOMOLOGICAL FUNCTORS*

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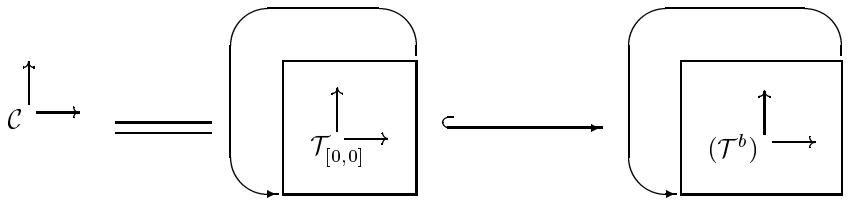
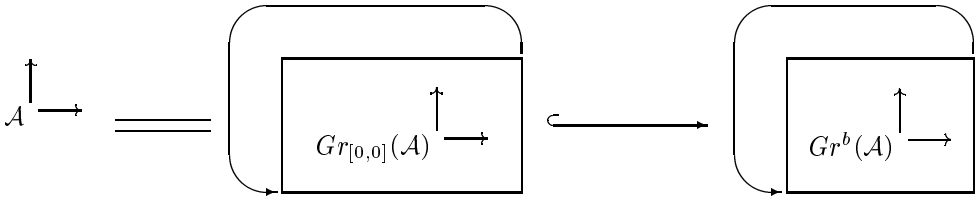
7. The beginning of the proof. The following theorems are true:

THEOREM 7.1. *Let $\mathcal{T}$ be a* t*–category. Let $\mathcal{C}$ be its heart. Then the inclusion*



induces a homotopy equivalence.

THEOREM 4.8. *Let $\mathcal{A}$ be an abelian category. The natural inclusion*



induces a homotopy equivalence.

In this article, we will give a complete proof of Theorem 4.8. In *K–theory for triangulated categories II* we will prove Theorem 7.1, in the special case $\mathcal{T} = D^b(\mathcal{A})$ with the usual t-structure. The general proof of Theorem 7.1 will have to wait until *K–theory for triangulated categories III*. Conceptually, the general proof of Theorem 7.1 is no harder; but the details are complicated. The proof involves many more homotopies, hence we leave it for a subsequent article.

The proof given for Theorem 7.1 (in the special case) will also provide a proof for Theorem 4.8. But first we will present another, simpler proof of Theorem 4.8. The simpler proof has two virtues. Firstly, it is simpler. Secondly, all but the very last step works for Construction 4.6, and the last step pinpoints just where we need the coherent choice of differentials. This second point is important. Because most of the proof is valid for Construction 4.6, we will be able to make certain deductions about the homotopy type of that construction.

The proofs of our two theorems begin the same way. In this section we will give the common beginning. Everything in this section is valid for all four constructions.

Beginning of the Proof. Because $\mathcal{T}^b = \bigcup_{m \le n} \mathcal{T}_{[m,n]}$, it will suffice to show that for $m' \le m \le n \le n'$, the inclusion $\mathcal{T}_{[m,n]} \subset \mathcal{T}_{[m',n']}$ induces a homotopy equivalence on whichever construction we work with. Clearly, it suffices to consider two cases:

7.1.1. $m' = m - 1$, $n' = n$;

7.1.2. $m' = m$, $n' = n + 1$.

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