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DIFFUSIVE LIMIT OF LATTICE GAS WITH MIXING CONDITIONS*

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Abstract. We prove, under certain mixing conditions, that the hydrodynamical limit of a stochastic lattice gas on the cubic lattice $\mathbb{Z}^d$ is governed by a nonlinear diffusion equation. Following [V1], we characterize the diffusion coefficient by a variational formula, which is equivalent to the Green-Kubo formula. The fluctuation-dissipation equation is established rigorously as an important step of the proof. Our mixing conditions are implied by the Dobrushin-Shlosman mixing conditions which are always valid at high temperature.

1. Introduction. Classical or Newtonian mechanics is described by a system of ordinary differential equations involving usually a two-body interaction potential. When the number of particles involved is large, rather than describe the motion of every particle, one usually tries to describe the collective behavior of these particles. The typical formulation involves the consideration of local averages of conserved quantities like density, average velocity, and energy. Since these local averages of conserved quantities tend to vary slowly in time, under some suitable space-time rescaling, that is referred to as hydrodynamical scaling, we expect to obtain a closed system of equations for these conserved quantities, as functions of macroscopic space and time. While such a transition is reasonably well understood from a physical point of view, these problems appear to be far beyond the current available methods for a rigorous mathematical treatment.

One way to make the problem tractable is to introduce some noise or randomness into the dynamics and in this way we obtain stochastic interacting particle systems. There are many interesting models that have been studied successfully and we wish to mention two of them. The first one is a Hamiltonian system perturbed by a weak noise considered in [OVY], that leads to the Euler equation under hydrodynamical scaling. The scaling there is the hyperbolic scaling $x \to \varepsilon x$ and $t \to \varepsilon^{-1} t$. The second model is one of interacting Brownian motions. Here the dynamics is controlled by the infinitesimal generator for the Markovian motion of these interacting Brownian particles. The infinitesimal generator involves a second order term which is the Laplace operator defining the noise, perturbed by a drift term or a first order term that comes from two particle interactions like in the Hamiltonian case. The similarity extends further in that the invariant measures are Gibbs measures very much like in the Hamiltonian case.

The natural scaling here is however the diffusive scaling $x \to \varepsilon x$, $t \to \varepsilon^{-2} t$ and the limiting equation is expected to be a nonlinear diffusion equation. This is rigorously

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