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# MIRROR PRINCIPLE I*

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**Abstract.** We propose and study the following *Mirror Principle:* certain sequences of multiplicative equivariant characteristic classes on stable map moduli spaces can be computed in terms of certain hypergeometric type classes. As applications, we compute the equivariant Euler classes of obstruction bundles induced by any concavex bundles – including any direct sum of line bundles – on $\mathbf{P}^n$. This includes proving the formula of Candelas-de la Ossa-Green-Parkes for the instanton prepotential function for quintic in $\mathbf{P}^4$. We derive, among many other examples, the so-called multiple cover formula for GW invariants of $\mathbf{P}^1$. We also prove a formula for enumerating Euler classes which arise in the so-called local mirror symmetry for some noncompact Calabi-Yau manifolds. At the end we interprete an infinite dimensional transformation group, called the mirror group, acting on Euler data, as a certain *duality group* of the linear sigma model.

## 1. Introduction.

**1.1. The Mirror Principle.** In section 2, we develop a general theory of Euler data, and give many examples. In particular we introduce the notions of a convex and a concave bundles on $\mathbf{P}^n$, and show that they naturally give rise to Euler data. In section 3 we apply our method to compute the equivariant Euler classes, and their nonequivariant limits, of obstruction bundles induced by a convex or a concave bundle.

We briefly outline our approach for computing multiplicative equivariant characteristic classes on stable map moduli. This outline also fixes some notations used later. Our approach is partly inspired by the idea of mirror transformations (Candelas et al), the idea of the linear sigma model (Witten, Morrison-Plesser, Jinzenji-Nagura,..), and the idea of using the torus action and equivariant techniques (Ellinsrud-Stromme, Kontsevich, Givental,..). All these ideas are syntheized with what we call the *Mirror Principle*, which we now explain.

Let $M$ be a projective manifold and $\beta \in H^2(M,\mathbf{Z})$. Let $\bar{\mathcal{M}}_{g,k}(\beta,M)$ be the stable map moduli space of degree $\beta$, arithmetic genus $g$, with $k$ marked points. For a good introduction to stable maps, see [16]. Throughout this paper, we shall only deal with the case with $g=0$.

We begin by analyzing two distinguished types of fixed points under an induced torus $T$ action on $\bar{\mathcal{M}}_{0,0}(d,\mathbf{P}^n)$. Both types of fixed points reflect the structure of the stable map moduli space. A smooth fixed point we consider is a degree $d$ cover of a $T$-invariant $\mathbf{P}^1$ joining two fixed points $p_i, p_j$ in $\mathbf{P}^n$. A singular fixed point we consider is in the compactification divisor. It is given by gluing together two 1-pointed maps $(f_1, C_1, x_1) \in \bar{\mathcal{M}}_{0,1}(r,\mathbf{P}^n)$ and $(f_1, C_2, x_2) \in \bar{\mathcal{M}}_{0,1}(d-r,\mathbf{P}^n)$ at the marked points with $f_1(x_1) = f_2(x_2) = p_i \in \mathbf{P}^n$ ($p_i$ being a $T$-fixed in $\mathbf{P}^n$), resulting in a degree $d$ stable map $(f,C)$. We consider two types of $T$-equivariant bundles $V$ on $\mathbf{P}^n$, which

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