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EXPANSION OF CONVEX HYPERSURFACES BY NONHOMOGENEOUS FUNCTIONS OF CURVATURE*

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1. Introduction. This paper is a sequel to our previous ones, Chow-Liou-Tsai [CLT] and Chow-Tsai [CT2], where we considered the expansion of a smooth closed uniformly convex hypersurface M_0 in Euclidean space along its outward normal vector direction with speed a function of the inverse of the harmonic mean curvature and the Gauss curvature respectively. Here we shall study the more general case concerning the expansion of a closed uniformly convex hypersurface M_0 of dimension n in Euclidean space $\mathbb{R}^{n+1}$ with the speed f a *nonhomogeneous* positive function of the principal radii. Needless to say, there have been many important works on the homogeneous flows. We refer the readers to the introductory remarks in the papers by Chow-Tsai [CT1], Chow-Liou-Tsai [CLT], or Andrews [A1,A2] for literature.

Let $X_0 : S^n \to \mathbb{R}^{n+1}$ be a smooth parametrization of $M_0 = X_0(S^n)$. The expansion is described by the equation

$$\begin{cases} \dfrac{\partial X}{\partial t} = f(\lambda_1, \lambda_2, \ldots.., \lambda_n) \cdot N \\ X(0) = X_0, \end{cases} \tag{1.1}$$

where $\lambda_1, \lambda_2, \ldots.., \lambda_n$ are the principal radii and N is the unit outward normal to the hypersurface $M_t := X_t(S^n)$. Here we assume $f \in C^\infty(\Gamma^+)$ is a positive symmetric real-valued function defined on the positive cone

$$\Gamma^+ = \{(x_1, x_2, ..., x_n) \in \mathbb{R}^n : \ x_i > 0, \ 1 \le i \le n\}$$

satisfying the strict parabolicity condition $\frac{\partial f}{\partial \lambda_i} > 0$ on Γ^+ for all $1 \le i \le n$. Because f is symmetric, the order of $\lambda_1, \lambda_2, ..., \lambda_n$ appearing in $f(\lambda_1, \lambda_2, ..., \lambda_n)$ is irrelevant. Under certain additional assumptions on the curvature function f analogous to those considered by Urbas [U1], et al, we prove that the evolving hypersurfaces remain smooth, strictly convex and expand to infinity while their shapes become round asymptotically. In particular, after an appropriate rescaling, the support functions of M_t converge to the constant 1 in C^1-norm.

The main observation of this paper is that we may remove the homogeneity assumption on f as imposed on the case of homogeneous expanding flows and still get results similar to (but weaker) the homogeneous case. However, the *concavity* assumption on f as required in the homogeneous expanding flows is still needed in order to go from $C^2(S^n \times [0,T))$ to $C^{2,\alpha}(S^n \times [0,T))$ estimate, after that we can quickly obtain all the higher derivatives estimates by parabolic bootstraps argument and hence establish the long time existence of the solution. We believe such concavity assumption is crucial here due to the lack of powerful a priori estimates for rather general nonlinear parabolic differential equations.

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