ASIAN J. MATH.
Vol. 3, No. 1, pp. 109–146, March 1999

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MIRROR PRINCIPLE II*

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Abstract. We generalize our theorems in *Mirror Principle I* to a class of balloon manifolds. Many of the results are proved for convex projective manifolds. In a subsequent paper, Mirror Principle III, we will extend the results to projective manifolds without the convexity assumption.

1. Introduction. For the long history of mirror symmetry, consult [17]. For a brief description of more recent development, see the introduction in [37][38]. The present paper is a sequel to *Mirror Principle I* [37]. Here, we generalize all the results there to a class of T-manifolds which we call balloon manifolds. These results were announced in [38].

Let X be a projective n-fold, and $d \in H^2(X, \mathbf{Z})$. Let $M_{0,k}(d, X)$ denote the moduli space of k-pointed, genus 0, degree d, stable maps $(C, f, x_1, .., x_k)$ on X [32]. Note that our notation is without the bar. By the work of [34](cf. [7][19]), each nonempty $M_{0,k}(d, X)$ admits a cycle class $LT_{0,k}(d, X)$ in the Chow group of degree $dim\ X + \langle c_1(X), d\rangle + n - 3$. This cycle plays the role of the fundamental class in topology, hence $LT_{0,k}(d, X)$ is called the virtual fundamental class.

Let V be a convex vector bundle on X. (ie. $H^1(\mathbf{P}^1, f^*V) = 0$ for every holomorphic map $f : \mathbf{P}^1 \to X$.) Then V induces on each $M_{0,k}(d, X)$ a vector bundle V_d, with fiber at $(C, f, x_1, .., x_k)$ given by the section space $H^0(C, f^*V)$. Let b be any multiplicative characteristic class [26]. (ie. if $0 \to E' \to E \to E'' \to 0$ is an exact sequence of vector bundles, then $b(E) = b(E')b(E'')$.) The problem we study here is to compute the characteristic numbers

$$K_d := \int_{LT_{0,0}(d,X)} b(V_d)$$

and their generating function:

$$\Phi(t) := \sum K_d\ e^{d\cdot t}.$$

There is a similar and equally important problem if one starts from a concave vector bundle V [37]. (ie. $H^0(\mathbf{P}^1, f^*V) = 0$ for every holomorphic map $f : \mathbf{P}^1 \to X$.) More generally, V can be a direct sum of a convex and a concave bundle. Important progress made on these problems has come from mirror symmetry. All of it seems to point toward the following general phenomenon [12], which we call *the Mirror Principle.* Roughly, it says that the function $\Phi(t)$ can be computed by a change of variables in terms of certain explicit special functions, loosely called generalized hypergeometric functions.

When X is a toric manifold with $c_1(X) \geq 0$, b is the Euler class, and V is a sum of line bundles, there is a general formula derived in [29] from mirror symmetry. This formula was later studied in [21] based on a series of axioms.

* Received March 22, 1999; accepted for publication March 24, 1999.
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