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RECOLLECTIONS ABOUT MY TEACHER, MICHAEL ATIYAH

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I am very fortunate to have been able to study with Michael Atiyah for a few months in 1968 and then for two more years (1969-71). His influence on me was especially strong at the beginning of my mathematical career, but his teaching has provided me with tools that I have constantly used throughout the years, even to this day. I am very grateful to him for this. For me he is not only a great mathematician and a great teacher, but also a human being of extraordinary generosity.

As an undergraduate at the University of Bucharest, I was very interested in topology and analysis on manifolds, so naturally, I came in contact with the work of Michael Atiyah. In fact, in 1965 we had a one semester course given by C. Teleman on the recently proved Atiyah-Singer index theorem, and I remember studying Henri Cartan's Paris seminar on the index theorem. I first saw Michael at the ICM in Moscow in 1966. He was sitting between two ladies (one was his wife, Lilly, the other was, he later told me, his mother) while Henri Cartan was talking about Michael's work for which he was just being awarded the Fields Medal. I also heard Michael's lecture at the ICM, which for me, was the high point of the Congress. But I first met Michael only two years later.

In the summer of 1968, I was at a summer school on pseudodifferential operators in Stresa, where Singer was giving one of the courses. There I talked with Singer (I think that he talked in English and I in French, since I didn't know any English) and told him that I was planning to go from Stresa to Warwick to a symposium on dynamical systems, although what I was really interested in was index theory; he then told me that he was in fact going to Oxford to work with Atiyah and why don't I come there too? So after a few weeks at Warwick, I went to Oxford. I remember very well my first meeting with Michael. He was in his office, at 25-29 St. Giles, with Singer. He asked me what problems I was interested in, and a few minutes later he explained to me what he and Singer were discussing: the problem of comparing the semicharacteristics of a $(4k+1)$-manifold with real or modulo 2 coefficients. He and Singer could prove that, if the real semicharacteristic was 0, then one can find two independent vector fields on the manifold, while E. Thomas could prove that if the manifold was spin and the mod 2 semicharacteristic was 0, then one can again find two independent vector fields. They naturally wanted to show that their result was stronger than that of E. Thomas, so they conjectured that the two semicharacteristics coincide for spin manifolds. During the following two months I stayed in Oxford and learned a lot of mathematics from Michael. I also found an answer to the question he asked about the semicharacteristic.

Before the two months were over, I received a letter from Deane Montgomery saying that, at the suggestion of Michael (who was about to move to Princeton), I was invited to spend a year at the IAS. In fact, I stayed at IAS for two years (1969-71). These two years were for me the equivalent of graduate school, with Michael as my teacher. But Michael was not only the most wonderful teacher one could have; he and Lilly were really like family to me during these years. I remember fondly the many times when I had meals in their home; on one occasion, when I developed a high fever, Michael drove me to Princeton Hospital so that a doctor can see me.

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During my stay at IAS, Quillen gave some lectures on his solution of the Adams conjecture in which a "Brauer lifting" of the standard modular representation of $GL_n(F_q)$ played a key role. After the lecture, I asked Michael, whether this Brauer lifting was explicitly known as a complex (virtual) representation of $GL_n(F_q)$. He told me that it was not known, except at the level of characters, by the work of J. A. Green. Somewhat later he asked me to read and explain to him a paper by S. Gelfand on discrete series representations of $GL_n(F_q)$. These were the seeds for my work (after moving to Warwick) on the Brauer lifting at the representation level, which led to my conversion to representation theory.

Sometime during my first year at IAS, I had the idea of twisting the signature operator on a compact manifold with a local system coming from a variable representation of the fundamental group into U(1). I felt that the resulting family of elliptic operators indexed by a torus must contain some interesting new information about the manifold. When I told Michael about this construction, he immediately said that this should have something to do with Novikov's higher signature. Eventually I proved that Michael's prediction was indeed correct.

Many years later, in may 1990, I met Michael (by that time he was Sir Michael) at a conference in Kyoto. After my second lecture (about canonical bases for quantized enveloping algebras of type A, D, E), Michael told me that quivers (which were used in my work) have also appeared in the work of Kronheimer, with two orientations considered simultaneously. The idea to use two orientations simultaneously turned out to be very useful in my subsequent work on the canonical basis.