ASIAN J. MATH.
Vol. 4, No. 1, pp. i–iii, March 2000

001

PREFACE TO KODAIRA'S ISSUE*

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Kunihiko Kodaira was born on March 16, 1915, the son of Gonichi Kodaira, a distinguished Japanese agriculturist who at one time held a cabinet post as vice-Minister of Agriculture and Forestry in the Japanese government. He received advanced degrees from the Department of Mathematics (1938) and Department of Physics (1941) of the University of Tokyo, and was an associate professor at the University of Tokyo from 1944 until 1951, where he received his Ph.D. in mathematics in 1949 by submitting a thesis on the subject of harmonic forms on Riemannian manifolds. In 1949, he came to the United States at the invitation of the Insitute for Advanced Study in Princeton, N.J., U.S.A. Following two years as a member of the Institute and a year as visiting professor at The Johns Hopkins University, he joined the Department of Mathematics of Princeton University in the autumn of 1952 (It was at exactly this time when the author of this preface, then a first-year graduate student at Princeton, began to follow Professor Kodaira's lectures.) Thereafter Kodaira divided his time on a half-and-half basis between Princeton University and the Institute for Advanced Study until 1961 when he accepted an invitation to visit Harvard University for one year. In the autumn of 1962, Kodaira joined the faculty of The Johns Hopkins University where he remained until 1965 when the Kodairas moved to Stanford University. They remained there until 1967 when Prof. Kodaira accepted a call to rejoin the faculty of the University of Tokyo, and went with his family to make a new home in the Nakaochiai section of Shinjuku-ku, Tokyo, where he resided for the rest of his life.

During his 18 year stay in the United States, Kodaira interacted with many other distinguished mathematicians to produce a prodigious quantity of mathematics of great beauty and profundity. Among these were W.L. Chow, F. Hirzebruch, L. Nirenberg, G. de Rham, and D.C. Spencer. Together Kodaira and Spencer produced at least twelve pioneering papers of the highest quality. Their relationship must surely be ranked as one of the most fruitful mathematical collaborations of all time.

In 1954, Kodaira shared the Fields Medals with J.P. Serre at the International Congress of Mathematicians held in Amsterdam that year. In 1957 Kodaira received the Japan Academy Prize and in the same year, the Cultural Medal, bestowed by the Emperor of Japan on only a very few Japanese citizens each year. Kodaira became a member of the Japan Academy in 1965.

Kodaira's earliest papers reflect a wide range of interests and were co-authored with several Japanese mathematicians of interests quite different from each other, including M. Abe, S. Iyanaga, and S. Kakutani.

After this initial period, Kodaira's work may be subdivided into several fields. Undoubtedly one of the most important influences on Kodaira's work was that of Hermann Weyl, whose work on solutions of partial differential equations using Hilbert space methods and the method of orthogonal projection which appear in Weyl's famous book, "Idee der Riemannschen Flächen" profoundly influenced Kodaira's work on harmonic differential forms on Riemannian manifolds, including as an important special case that of algebraic varieties. These ideas appear quite clearly in his earlier

*Received April 29, 2000.
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joint work with Spencer on the application of sheaves and their associated cohomology groups on algebraic varieties to algebraic geometry.

In fact, the subject of algebraic geometry is at the center of much of Kodaira's work, including joint work with W.L. Chow and with D.C. Spencer. But what makes this work so powerful and fruitful is its effective use of the methods of differential equations and regularity theorems, and the use of the parametrix to prove regularity theorems for solutions of partial differential equations, as well as the cohomology of complex manifolds with coefficients in coherent analytic sheaves. These tools from analysis had not been used effectively in most of the earlier work, for example of the Italians, in algebraic geometry. Their applications provided a real break- through in advancing this subject. Kodaira and Spencer and some of their collaborators and colleagues in the same field, such as M. Kuranishi and L. Nirenberg applied the same kinds of ideas in connection with other structures that could be superimposed on Riemannian manifolds, complex manifolds, and algebraic varieties. But the particular case of compact complex manifolds, and especially that of smooth complete complex algebraic varieties, with their richer structures and having the added advantage that the cohomology groups with coefficients in coherent analytic sheaves are finite dimensional, afforded a fertile field for developing sharp and clear results in this area.

Kodaira's work "On a differential-geometric method in the theory of analytic stacks", inspired by work of S. Bochner, has proved, with its application to vanishing theorems, to be an extremely useful criterion for the existence of non-constant meromorphic functions (when they exist) on compact complex manifolds, as Kodaira himself has remarked. This circle of ideas has deep implications for the work on deformations of complex structures.

The work on deformations of complex structures done by Kodaira and Spencer is perhaps their most profound in this area and suggests some most interesting questions that have yet to be explored. In the preface to his book mentioned below, Kodaira ascribes the inspiration for this joint work to results obtained by Fröhlicher and Nijenhuis in "A theorem on stability of complex structures" (Proc. Nat. Acad. Sci., U.S.A., 43(1957), 239-241), which Kodaira and Spencer also call the starting point of their large joint work. This truly pioneering work, "On deformations of complex structures, I-III", remains as a cornerstone of work accomplished up to now, and as the likely basis of some truly interesting progress in the future. It starts from the concept of variation of complex structure on a complex manifold V as measured by the first cohomology group $H^1(V, \Theta_V)$, where Θ_V is the sheaf of germs of holomorphic vector fields on V. This cohomology group is in many cases to be viewed as the tangent space to the moduli space of deformations of V, and its dimension in such cases is the number of moduli or effective parameters on which depend the complex structures on the underlying C^∞ manifold. These authors have provided criteria for these conclusions to be valid, for example the vanishing of the corresponding 0-th and 2-nd cohomology groups of V with coefficients in T_V. An excellent treatment of these topics in essentially textbook form is Kodaira's own book on the subject, published in 1986, "Complex Manifolds and Deformation of Complex Structures", which holds a virtually self-contained treatment of the subject of differential forms, harmonic forms, Hodge theory, etc.,as well as of the necessary regularity theorems in an appendix by Daisuke Fujiwara. This book appeared in the very year (1986) in which it was decided that the 1990 sessions of the International Congress of Mathematicians would be held in Kyoto, Japan.

A striking characteristic of Kodaira's work in general, as particularly exemplified in this book, is his use of explicit, down-to-earth, concrete calculations and constructions. The author of this preface was very much impressed by this feature of Kodaira's style upon attending Kodaira's lectures in Princeton, as alluded to earlier.

One must not at all think of Kodaira's time in Tokyo as a retirement from active mathematics. Indeed, the last 30 years of his life were very active. He organized a large group of advanced students and faculty of the University of Tokyo and other universities into a seminar which met regularly and weekly, usually on Saturdays, to have talks presented by members of the seminar and others on subjects of current interest, mostly in algebraic geometry in areas close to Kodaira's work. In 1975, because of retirement-age rules, Kodaira had to leave Tokyo University, after which he joined the faculty of Gakushuin University, the so-called "Peers' School" where many persons of high social standing, including the Royal Family of Japan, went for a higher education. Nevertheless, Kodaira was still active in mathematics and received frequent visits from students and friends, including S. Iitaka, T. Shioda, the author of this preface, and others. He continued very lively and maintained an active interest in mathematics and in other things. And in 1986 there was published Kodaira's book mentioned above.

Indeed, when in 1986, Kyoto, Japan, was selected as the site for the 1990 meeting of the International Congress of Mathematicians, Prof. Kodaira accepted appointment as chairman of the organizing committee for the Congress. In that capacity, he was very active in soliciting financial support for the Congress from prominent business men, many of whom were former university classmates of Kodaira. However, it seemed to the author of this preface that the very intensity of Kodaira's work and activity at this period may well have precipitated a decline in his health and well-being. He had never relished official and bureaucratic duties, perhaps because of his retiring and modest nature, as evidenced by his unhappiness with his duties as Dean of Sciences at the University of Tokyo. At any rate, from 1986 to 1990 his health underwent a decline, so that even as chairman of its organizing committee he was unable to be present at the meeting of the ICM in Kyoto in 1990. And this general deterioration in his health progressed until his death on July 26, 1997. The author of this preface was most pleased and honored to have enjoyed the privilege of Professor Kodaira's acquaintance and friendship over a period of nearly 45 years.