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ON THE NEGATIVITY OF KERNELS OF KODAIRA-SPENCER MAPS ON HODGE BUNDLES AND APPLICATIONS*

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Let X denote a smooth complex projective variety, and $S = \sum_i S_i \subset X$ a normal crossing divisor with smooth components. We consider a polarized variation of Hodge structure $\mathbb{V}_0$ on $X_0 = X \setminus S$ and the period map

$$\phi : X_0 \to \mathcal{D}/\Gamma$$

corresponding to $\mathbb{V}_0$.

THEOREM 0.1. *Suppose that ϕ is injective at some points. Let $D \subset X_0$ denote the degeneration locus of ϕ. Then*

i) *$\Omega^1_X(logS)$ is weakly positive on $X_0 \setminus D$ in the sense of Viehweg [V1], [V3]. i.e. for any ample line bundle L on X and for any positive number α there exists some positive number β such that the symmetric tensor*

$$S^{\alpha\beta}(\Omega^1_X(logS)) \otimes L^{\beta}|_{X_0\setminus D}$$

is generated by global sections on X.

ii) *the Kodaira dimension of $\det \Omega^1_X(logS)$ is equal to $\dim X$. i.e. (X, S) is of logarithmic general type.*

Viehweg recently asked that if the cotangent bundles of moduli varieties of polarized varieties with log-pole along infinity are positive in some sense. It is well known that the moduli space of curves of genus $g \geq 1$ has the semi-positivity property. Theorem 0.1 shall give a positive answer to this question in the case when some Torelli map on the universal family is locally injective. It is extremely very interesting to know if that will still be true without existence of locally injective Torelli maps at all!

In particular, Theorem 0.1 recovers the following two special cases, which were known previously. Let $X_0 =$ an arithmetic variety $\mathcal{D}/\Gamma$, and X is a smooth compactification of X_0. Then Mumford [M] proved that (X, S) is of logarithmic general type by using automorphic forms, which was studied in [BB]. Recently, Luo [L] showed that the moduli spaces of polarized Calabi-Yau manifolds are of log general type. Luo used the method in [SY] and the general Schwarz lemma of Yau [Y]. Our method here is essentially semi-positivity of $\Omega^1_X(\log S)$, and Kawamata-Viehweg vanishing theorem. The author believes that one can also use the methods of [SY] and [Y] to prove ii).

Inspired by a recent paper of Bedulev-Viehweg [BV] Jost and the author [JZ] recently proved similar inequalities, the so called Arakelov type inequalities for Hodge bundles over curves (also see [P2]). The inequalities states that the degree of Hodge bundles are bounded above by the degree of the log-canonical bundles of the curve C multiplies some constants, which depends on the Hodge type of E only, i.e.

$$\deg E^{p,q} \leq r^{p,q}(2g(C) - 2) + \#S).$$

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