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DEFORMING TRANSVERSE RIEMANNIAN METRICS OF FOLIATIONS*

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Abstract. The Ricci flow of Riemannian metrics on a compact manifold can be interpreted as a deformation of Cartan connections of hyperbolic type. The purpose of this paper is to show that there is analogous interpretation for the holonomy invariant transversal Riemannian metric defined for foliations over compact manifolds. Once the short-time existence of the transversal flow of Cartan connections is established, the theorems proved using deformation of the metric through the Ricci flow are generalized to the case of a Riemannian foliation on a compact Riemannian manifold.

1. Introduction and Results. In [9] the Ricci flow of Riemannian metrics on a compact manifold was interpreted as a deformation of Cartan connections of hyperbolic type. The purpose of this paper is to show that there is analogous interpretation for the holonomy invariant transversal Riemannian metric defined for foliations over compact manifolds.

As the first consequence, using the theorem of Hamilton [3, section 5] we obtain the short-time existence of the transversal flow of Cartan connections on M. Following the idea of Bemelmans, Min-Oo and Ruh introduced in [2] and using the maximum principle as in [1] we obtain the following smoothing property:

THEOREM 1.1. *Let $\mathcal{F}$ be a foliation of codimension q defined on a compact n-dimensional Riemannian manifold M with a holonomy invariant transversal Riemannian metric g_Q on the normal bundle Q. There exists a smooth curve of transversal Riemannian metrics $g(t)$ with $g(0) = g_Q$ and universal constants $C(q)$ and $\epsilon(q)$ depending only on the codimension q, such that*

$$\sum_{k=0}^{\infty} \frac{t^k}{((k+1)!)^2} |D^k F^D(g(t))|^2 \leq C(q)|F^D(g(0))|^2 \tag{1.1}$$

uniformly for $t \in [0, \epsilon(q)]$, where $|D^k F^D(g(t))|$ is the supremum of the $k-$th covariant derivative of the transversal curvature tensor $F^D(g(t))$ of the metric $g(t)$.

Theorems proved using deformation of the metric through the Ricci flow (see [7, 8, 12]) can now be generalized to the case of a Riemannian foliation on a compact Riemannian manifold. In particular, we obtain the following theorems (see [3, 4]):

THEOREM 1.2. *Let $\mathcal{F}$ be a foliation of codimension three on a compact manifold M with a holonomy invariant transversal Riemannian metric g_Q for the normal bundle Q. Suppose that the Ricci curvature of g_Q is positive definite. The metric g_Q can be deformed to a holonomy invariant transversal Riemannian metric with constant positive sectional curvature.*

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