ASIAN J. MATH.
Vol. 4, No. 2, pp. 391–436, June 2000

010

MODULI SPACES OF $PU(2)$-MONOPOLES*

ANDREI TELEMAN†

1. Introduction. The most natural way to prove the equivalence between Donaldson theory and Seiberg-Witten theory is to consider a suitable moduli space of "non-abelian monopoles". In [OT5] it was shown that an S^1-quotient of a moduli space of quaternionic monopoles should give an homological equivalence between a fibration over a union of Seiberg-Witten moduli spaces and a fibration over certain $Spin^c$-moduli spaces [PT1].

By the same method, but using moduli spaces of $PU(2)$-monopoles instead of quaternionic monopoles, one should be able to express any Donaldson invariant in terms of Seiberg-Witten invariants associated with the twisted abelian monopole equations of [OT6]. In [T1], [T2], we have shown that this idea can be further generalized to express Donaldson-type invariants associated with higher symmetry groups in terms of new Seiberg-Witten-type invariants.

The strategy has a very general algebraic-geometric analogon, which we call the "Master Space" strategy. This procedure, developed by Ch. Okonek and the author [OT7], [OST] reduces the problem of the computation of certain numerical invariants of a GIT moduli space to similar computations on simpler moduli spaces. One "couples" the given GIT problem to a simpler one (having the same symmetry group), and then studies the "Master Space" associated with the coupling as a $\mathbb{C}^*$-space. The fixed point locus of the $\mathbb{C}^*$-action consists of the original moduli space and a union of simpler ones. Then one can use the S^1-quotient of the master space to define a homological equivalence between a projective fibration over the initial moduli space and a projective fibration over the other components of the fixed point locus. In the GIT-framework, as in the gauge theoretical one, the technical difficulty is the same: the master space can be singular. The present paper deals with this difficulty in the gauge theoretical situation.

A program for proving the equivalence between Donaldson theory and Seiberg-Witten theory, which also uses moduli spaces of non-abelian monopoles, is due to Pidstrigach and Tyurin [PT2], and was already announced by Pidstrigach in a Conference at the Newton Institute in Cambridge, in December 1994.

There are, however, several important differences between Pidstrigach-Tyurin's original approach, and the strategy developed by Ch. Okonek in collaboration with the author, which is the strategy we follow in the present paper.

First, our equations have a gauge group of the form $SU(E)$ and hence the moduli spaces which we construct are S^1-spaces; in contrast, the Pidstrigach-Tyurin equations [PT2] have a gauge group of the form $U(E)$. Whereas we fix the connection in the determinant line bundle, they only fix the curvature of this connection. If $H_1(X,\mathbb{Z}) = 0$, their moduli space is the S^1-quotient of ours. On the other hand, the S^1-operation plays a very important role in our strategy: The description of the ends around the abelian locus at infinity uses in an essential way the S^1-equivariance of the local models.

*Received April 02, 1999; accepted for publication May 4, 1999.
†CMI, Université de Provence, 39, Rue F. Joliot Curie, 13453 Marseille Cedx 13, France (teleman@cmi.univ-mrs.fr) and Faculty of Mathematics, University of Bucharest, Romania.