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THE MONODROMY PAIRING*

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For an Abelian variety over a complete local field K, let A' denote its dual and $\Gamma(A)$ its "fundamental group" (see §1 below). In SGA7I, Exposé IX §9, Grothendieck defined a pairing between $\Gamma(A)$ and $\Gamma(A')$ with values in $\mathbf{Z}$, if A has semi-stable reduction, which he called the monodromy pairing. In fact, he proved a p-adic analogue of the Picard-Lefshetz formula when A is a Jacobian of a curve with semi-stable reduction (see §5 below).

Raynaud [R1] also wrote down a definition of a pairing between $\Gamma(A)$ and $\Gamma(A')$ when A has semi-stable reduction using bi-extensions and Grothendieck asserted that Raynaud's pairing is the same as his in Chapter IX, §14.2.5 of SGA7 but gave no details of a proof. We provide a proof in §2 based on Werner's analysis of Raynaud's pairing which uses an observation of Reversat-Van der Put [R-vP].

This result was needed to show, in [CI], that the monodromy operator on the first de Rham cohomology group of a semi-stable curve defined in [pSI] is the same as that of Hyodo-Kato [HK] and it was also used in §3 of [W] to study local heigfht pairings.

The remainder of the paper is a rigid analytic proof of the p-adic Picard-Lefshetz formula based on the aforementioned expression of Raynaud's formula for the monodromy pairing. A sketch of an alternate proof of this result is contained in [FC], Chapter III, Theorem 8.3.

The main new technical result is an explicit relationship, proven in §4, between the rigid residue maps from the regular differentials on a curve with semi-stable reduction over K to K, defined in Reciprocity Laws on Curves [C2], and rigid homomorphisms from $\mathbf{G}_m$ into the Jacobian of the curve.

We first prove, in §5, the Picard-Lefshetz formula in the case of the Jacobians of Mumford curves. Although this is not essential to our ultimate general proof, it provides motivation and in fact we are able to prove more in this case.

In the following, $K \subset \mathbf{C}_p$ will be a finite extension of the completion of the maximal unramified extension of $\mathbf{Q}_p$, R will be its ring of integers, k will be its residue field and π will be a uniformizing parameter of K. We will let v be the valuation on $\mathbf{C}_p$ such that $v(\pi) = 1$. All objects discussed below will be supposed defined over K, unless otherwise indicated.

Generalizing the notions of [C2]: If F is a complete subfield of $\mathbf{C}_p$, by a **discoid** space over F we mean an affinoid which becomes isomorphic to a finite union of disjoint affinoid disks after a finite base extension and by a **wide open space** over F we mean the complement in a smooth complete curve of a discoid subdomain. If W is a wide open space over F the set of **ends** of W is the inverse limit of $CC(W_{\mathbf{C}_p} - X)$ where X runs over the affinoid subdomains of $W_{\mathbf{C}_p}$. The group $Gal(\mathbf{C}_p/F)$ clearly acts on this set. By a **basic wide open pair**, we mean a pair (W, X), where W is a wide open and X is an affinoid in W with good reduction such that after a finite extension the connected components of $W - X$ are annuli and the map from $CC(W - X)$ to the ends of W is a bijection. A wide open which is a member of a basic wide open pair will be called a **basic wide open**. The results and proofs of [C2] carry over easily to this more general context.

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