ASIAN J. MATH.
Vol. 4, No. 2, pp. 337–344, June 2000

004

ALGEBRAIC K-THEORY OF PURE BRAID GROUPS*

C. S. ARAVINDA†, F. T. FARRELL‡, AND S. K. ROUSHON§

Abstract. In this article we introduce the notion of strongly poly-free groups and show that the Whitehead group of any such group vanishes. We also show that the pure braid groups are strongly poly-free.

0. Introduction. A well-known conjecture in Geometry and Topology states that the Whitehead group of any finitely presented torsion free discrete group vanishes. So far this has been checked for many classes of such groups, e.g. free group [ST1], free abelian group [BHS], fundamental group of closed nonpositively curved Riemannian manifolds [FJ1]. There is a recent extension of the last result to complete nonpositively curved Riemannian manifold with some regularity condition on the sectional curvature [FJ3].

Now given any finitely presented torsion free group, to check the vanishing of the Whitehead group for this group, at first one would like to check if this group is the fundamental group of a complete nonpositively curved Riemannian manifold. We consider the case of pure braid group. It is not yet known if these groups can even act faithfully as a group of motions on a simply connected complete nonpositively curved Riemannian manifold. In this paper we introduce a new class of groups, namely the *strongly poly-free* groups which is a proper subclass of the class of poly-free groups and prove that the Whitehead group vanishes in this class of groups. We also show that the pure braid groups are strongly poly-free. We use some geometric results on 3-manifolds which fiber over the circle, the Fibered Isomorphism Theorem of Farrell and Jones and an induction argument to prove the result.

The paper is organized in the following way. In Section 1 we define strongly poly-free groups and state and prove that the Whitehead group of these groups are trivial. Section 2 is devoted to proving that the pure braid groups are strongly poly-free. We prove that the Whitehead group of the pure braid group of any compact (connected) surface except for the 2-sphere and the projective 2-plane vanishes in Section 3.

ACKNOWLEDGMENT. This research was supported in part by the National Science Foundation and by the United Nations Educational, Scientific and Cultural Organization. The project crystallized while the authors were visitors at the Abdus Salam International Center for Theoretical Physics. They wish to thank ICTP for the kind invitation and hospitality which greatly facilitated the completion of this research. Part of this work was done during the last author's stay in the Max-Planck Institut für Mathematik. He would like to thank the institute for the hospitality.

1. A vanishing criterion and strongly poly-free groups. This section is devoted to defining the notion of strongly poly-free group and to proving that the Whitehead group of these groups vanishes.

* Received March 4, 1999; accepted for publication May 31, 1999.

† SPIC Mathematical Institute, 92, G. N. Chetty Road, Chennai 600 017, India (aravinda@smi.ernet.in).

‡ Department of Mathematics, SUNY, Binghamton, NY 13902, USA.

§ School of Mathematics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India (roushon@math.tifr.res.in).