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A REMARK ON DIVISORS OF CALABI-YAU HYPERSURFACES*

LIH-CHUNG WANG†

Abstract. We prove that a non-singular hypersurface of degree $\geq n+1$ in $\mathbb{P}^n$ for $n \geq 4$ does not contain a reduced irreducible divisor which admits a desingularization having nef anticanonical bundle.

1. Introduction. In this paper we shall generalize a theorem [1] of Chang and Ran to the higher dimensional case. They proved that a generic hypersurface of degree ≥ 5 in $\mathbb{P}^3$ or $\mathbb{P}^4$ does not contain a reduced irreducible divisor which admits a desingularization having numerically effective anticanonical bundle. The $\mathbb{P}^3$ case is a conjecture of Harris which is first proven by G. Xu [5] with a different method. The natural generalization of their theorem is the nonexistence of a divisor with numerically effective (nef) anticanonical bundle on a generic hypersurface of degree $\geq n+1$ in $\mathbb{P}^n$ for $n \geq 5$ (See Corollary 3.3). However, the interesting case is the case of Calabi-Yau hypersurfaces (degree equal to $n+1$) since G. Xu gave a geometric genus bound for divisors on generic hypersurfaces of general type. In fact, our setup in this paper is a little more general. We prove that a non-singular complete intersection in Grassmannian with a similar degree assumption does not contain a reduced irreducible divisor which admits a desingularization having numerically effective anticanonical bundle.

Let us fix notations in this paper. Thus let X be a non-singular complete intersection of type $(m_1, m_2, \cdots, m_k)$ in Grassmann variety $G(r, n+1)$ such that $\dim X \geq 3$ and $m = m_1 + m_2 + \cdots + m_k \geq n+1$, and suppose $\bar{D} \subset X$ is an irreducible and reduced divisor. Let $f : D \longrightarrow \bar{D} \subset X$ be a desingularization, l denote the dimension of D and L denote $f^*\mathcal{O}_G(1)$. Obviously, L is nef and big. Let K_D be the canonical bundle of D. Let S and Q be the universal subbundle and universal quotient bundle on $G(r, n+1)$. $Q^\vee$ denotes the dual of Q.

The main technical statement we are going to prove is the following.

PROPOSITION 1.1. *A non-singular complete intersection X of type $(m_1, m_2, \cdots, m_k)$ in Grassmann variety $G(r, n+1)$ such that $m = m_1 + m_2 + \cdots + m_k \geq n+1$ does not contain a reduced irreducible divisor which admits a desingularization having $H^0(K_D \otimes f^*Q^\vee) = 0$ and $H^1(K_D - m_i L) = 0$ for $i = 1, \cdots, k$.*

Here we review the definition and some basic properties of reflexive sheaves (See [3]). Let $\mathcal{F}^{\vee\vee}$ be the double dual of $\mathcal{F}$. A coherent sheaf $\mathcal{F}$ is reflexive if the natural map $\mathcal{F} \longrightarrow \mathcal{F}^{\vee\vee}$ is an isomorphism. Define the singularity set of $\mathcal{F}$ to be the locus where the $\mathcal{F}$ is not free over the local ring.

It is well-known that the singularity set of a torsion-free sheaf on D is at least 2-codimensional. Moreover, the singularity set of a reflexive sheaf on D is at least 3-codimensional. It is also well-known that, in general, any reflexive rank 1 sheaf on an integral and locally factorial scheme is invertible.

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†Department of Applied Mathematics, National Donghwa Universitym Shoufeng, Hualien 974, R.O.C. (lcwang@server.am.ndhu.edu.tw). Paper partially supported by National Science Council (NSC-88-2115-M-259-001).