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FINITE-DIMENSIONAL FILTERS WITH NONLINEAR DRIFT XIII: CLASSIFICATION OF FINITE-DIMENSIONAL ESTIMATION ALGEBRAS OF MAXIMAL RANK WITH STATE SPACE DIMENSION FIVE*

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Abstract. The idea of using estimation algebras to construct finite dimensional nonlinear filters was first proposed by Brockett and Mitter independently. For this approach, one needs to know explicitly the structure of these estimation algebras in order to construct finite dimensional nonlinear filters. Therefore Brockett proposed to classify all finite dimensional estimation algebras. Chiou and Yau [ChYa1] classify all finite dimensional estimation algebras of maximal rank with dimension of the state space less than or equal to two. The purpose of this paper is to give a new result on classification of all finite dimensional estimation algebras of maximal rank with state space dimension less than or equal to five.

1. Introduction. The idea of using estimation algebras to construct finite dimensional nonlinear filters was first proposed by Brockett [Br] and Mitter [Mi] independently. The advantage of this finite-dimensional nonlinear filter is at least the same as Kalman-Bucy filter. Moreover, it avoids the disadvantages of Kalman-Bucy filter such as Gaussian initial condition as well as linearity assumption of the drift term. For more detail, we refer the readers to [TWY] and [Ya], in which the links between finite dimensional estimation algebras and finite dimensional filters were discussed. It is clear from the works of [TWY] and [Ya] that one needs to know explicitly the structure of these estimation algebras in order to construct finite dimensional nonlinear filters. In 1983, Brockett proposed to classify all finite dimensional estimation algebras in his talk at the International Congress of Mathematics. If the drift term of the nonlinear filtering system has a potential function (i.e. drift term is a gradient vector filed), then the corresponding estimation algebra is called exact. In [TWY], Tam, Wong and Yau have classified all finite dimensional exact estimation algebras of maximal rank with arbitrary state space dimension. In [ChYa1], Chiou and Yau are able to classify all finite dimensional estimation algebras of maximal rank with state space dimension less than or equal to two. The novelty of their theorem is that there is no assumption on the drift term of the nonlinear filtering system. In [CYL1], Chen, Leung and Yau classify all finite dimensional estimation algebras of maximal rank with state space dimension equal to 3 (without any assumption on the drift term). They introduced a new matrix equation in [CYL2] and showed that this matrix equation has only trivial solution if the state space dimension is at most four. They reduced the classification problem of finite dimensional estimation algebras with maximal rank to the problem of nonexistence of nontrivial solution of this new matrix equation by using the fact that η_4, homogeneous degree four part of η, depends only on $x_{k+1},\ldots,x_n$ where k is the quadratic rank of the estimation algebra. Recently Wu, Yau and Hu [WYH] have a direct proof of nonexistence of nontrivial solution of this new matrix equation. In this paper, we develop a completely different technique than [CYL2]. Moreover we

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