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013

A NUMERICAL BOUND FOR SMALL PRIME SOLUTIONS OF SOME TERNARY LINEAR EQUATIONS, II*

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1. Introduction. Since Vinogradov's remarkable result [V] that every sufficiently large odd integer can be represented as a sum of three primes in 1937, there are many results on the solvability to the Diophantine linear equation

$$p_1 + p_2 + p_3 = b \tag{1.1}$$

with prime variables. Hua's result[Hu1] is more general than the others; he proved in 1938 that every sufficiently large odd integer b can be written as the form

$$p_1 + p_2 + p_3^k = b \tag{1.2}$$

where $k \geq 1$ is a fixed integer. Then Hua established in [Hu2] the additive theory of prime numbers based on a series of generalizations of the works of Vinogradov on prime number theory. Alan Baker[B] took a step further and considered in 1967 the problem on bounds for small prime solutions of the linear equation

$$a_1p_1 + a_2p_2 + a_3p_3 = b \tag{1.3}$$

where a_1, a_2, a_3 are integers such that

$$a_1a_2a_3 \neq 0 \quad \text{and} \quad (a_1,\, a_2,\, a_3) = 1, \tag{1.4}$$

and b is any integer satisfying

$$b \equiv a_1 + a_2 + a_3 (\operatorname{mod} 2) \quad \text{and} \quad (b,\, a_i,\, a_j) = 1 \quad \text{for} \quad 1 \leq i < j \leq 3. \tag{1.5}$$

Put

$$A := \max\left\{|a_1|\,,\, |a_2|\,,\, |a_3|\right\}. \tag{1.6}$$

As the culmination of a series of earlier discoveries[L1], [L2] on the bound of the small prime solutions to equation (1.3), it was proved [LT1]:

THEOREM 0. *Assume the conditions of (1.4) and (1.5) on a_j's and b.*

If all the a_j's are positive, then there is a computable absolute constant $V > 0$ such that equation (1.3) is solvable in prime variables p_1, p_2 and p_3 when $b \gg A^V$;

If all the a_j's are not of the same sign, then there is a computable absolute constant $B > 0$ such that equation (1.3) has a prime solution p_1, p_2, p_3 with

$$\max\left\{|a_1|\,p_1,\, |a_2|\,p_2,\, |a_3|\,p_3\right\} \ll \max\left\{|b|,\, A^B\right\}.$$

Here $\gg$ and $\ll$ are the Vinogradov symbol and the implied constants in the symbols are computable absolute constants.

As indicated in [LT1, §1] and [LT2, Remark 1.1 and Remark 2.1], conditions (1.4) and (1.5) are necessary and Theorem 0 implies Linnik's theorem that the least prime in any arithmetic progression $\{nq + \ell\}_{n=1,\,2,\cdots}$ with $(q,\, \ell) = 1$, is $\ll q^L$, where

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