Asian J. Math.
Vol. 5, No. 1, pp. 161–168, March 2001

010

KNOT CONCORDANCE AND TORSION*

CHARLES LIVINGSTON† AND SWATEE NAIK‡

1. Introduction. The classical knot concordance group, $\mathcal{C}_1$, was defined in 1961 by Fox [F]. He proved that it is nontrivial by finding elements of order two; details were presented in [FM]. Since then one of the most vexing questions concerning the concordance group has been whether it contains elements of finite order other than 2–torsion. Interest in this question was heightened by Levine's proof [L1, L2] that in all higher odd dimensions the knot concordance group contains an infinite summand generated by elements of order 4. In our earlier work studying this problem we proved the following [LN]:

Theorem 1.1. *Let K be a knot in S^3 with 2–fold branched cover M_K. If the order of the first homology with integer coefficients satisfies $|H_1(M_K)| = pm$ with p a prime congruent to 3 mod 4 and gcd(p, m) = 1, then K is of infinite order in the classical knot concordance group, $\mathcal{C}_1$.*

An immediate corollary was that all of the prime knots of less than 11 crossings that are of order 4 in the algebraic concordance group are of infinite order in the concordance group. There are 11 such knots [M]. One simple case of a much deeper corollary states that if the Alexander polynomial of a knot satisfies $\Delta_K(t) = 5t^2 - 11t + 5$ then K is of infinite order in $\mathcal{C}_1$. According to Levine [L2], any higher dimensional knot with this polynomial is of order 4 in concordance.

Here our goal is to prove the following enhancement of the theorem stated above:

Theorem 1.2. *Let K be a knot in S^3 with 2–fold branched cover M_K. If $H_1(M_K) = Z_{p^n} \oplus G$ with p a prime congruent to 3 mod 4, n odd, and p not dividing the order of G, then K is of infinite order in $\mathcal{C}_1$.*

As we will describe below, the significance of this result goes beyond its apparent technical merit; however, even in terms of computations it is an important improvement. Let H_p denote the p–primary summand of $H_1(M_K)$.

Corollary 1.3. *Let n be a positive integer such that some prime p congruent to 3 mod 4 has odd exponent in the prime power factorization of $4n + 1$. Then a knot K with Alexander polynomial $nt^2 - (2n + 1)t + n$ and H_p cyclic is of infinite order in the concordance group.*

Note that according to Levine [L2], any such knot represents an order 4 class in the algebraic concordance group. The n–twisted doubles of knots provide infinitely many examples of knots with Alexander polynomial $nt^2 - (2n + 1)t + n$ and $H_1(M_K)$ cyclic. Further details and examples will be provided in the last section.

Casson and Gordon's first examples of algebraically slice knots that are not slice [CG1, CG2] were taken from the set of twisted doubles of the unknot. Our analysis

* Received April 8, 2000; accepted for publication April 19, 2000.
† Department of Mathematics, Indiana University, Bloomington, IN 47405, U.S.A. (livingst@indiana.edu).
‡ Department of Mathematics, University of Nevada, Reno, NV 89557, U.S.A. (naik@unr.edu).