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002

GENERIC MODULES FOR EXTENSION ALGEBRAS*

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Abstract. Let A be a tame hereditary algebra (finite-dimensional over an algebraically closed field), R_A^m $(m \geq 1)$ the extension algebra of A. A generic R-module M over an arbitrary ring R is by definition an indecomposable R-module of infinite length, such that M considered as an $\mathrm{End}(M)$-module, is of finite length (its endolength). In this paper we investigate the generic modules of $\widehat{A}$ (the repetitive algebra of A) and R_A^m. It is proved that R_A^m has at least $2m$ generic modules.

Introduction. The notion of generic module was introduced in [1] by Crawley-Boevey. The concept seems to be quite natural and important. The generic modules even have a dominating position in the category of modules. In [2], it was shown that whether a finite-dimensional algebra over an algebraically closed field is tame or wild is determined completely by the behaviour of the generic modules for that algebra.

In [3], Aronszajn and Fixman gave the concept of a divisible module for the Kronecker algebra and showed that for the Kronecker algebra there exists a unique indecomposable torsion-free divisile module. In [4], Ringel generalized the work of Aronszajn and Fixman and proved the same result for a tame hereditary algebra. Ringel's work, in fact, showed that for a tame hereditary algebra, there exists a unique generic module. In [6], we solved the existence and uniqueness of generic module for the tilted algebra determined by a tame hereditary algebra.

Following [1], A generic R-module M over an arbitrary ring R is by definition an indecomposable R-module of infinite length, such that M considered as an $\mathrm{End}(M)$-module, is of finite length (its endolength). Of course, the generic modules with endomorphism ring a division ring just, form the vertices of the (Cohn) spectrum of R. By [1], the endomorphism ring of a generic module always is a local ring.

Our purpose here is to investigate the generic module of the extension algebra R_A^m (defined below) for a tame hereditary algebra A. In section 1, we investigate the ν-orbits of generic modules for a repetitive algebra. we shall prove that $\mathrm{Mod}\,\widehat{A}$ has at least two ν-orbits of generic $\widehat{A}$-modules (Theorem 1.2). In section 2, we shall prove our main result on generic modules of R_A^m: R_A^m has at least $2m$ generic modules (Theorem 2.4 and Corollary 2.5).

Throughout this paper, we denote by k an algebraically closed field. An algebra means basic, connected and finite-dimensional k-algebra. For an algebra A we denote by $\mathrm{Mod}\,A$ the category of all right A-modules, by $\mathrm{mod}\,A$ the full subcategory of $\mathrm{Mod}\,A$ consisting of all finitely generated right A-modules and by $\underline{\mathrm{mod}}\,A$ the corresponding stable category. We shall use freely properties of the Auslander-Reiten sequences, irreducible maps, Auslander-Reiten translation $\tau = D\mathrm{Tr}$ and $\tau^{-1} = \mathrm{Tr}D$, and the Auslander-Reiten quiver Γ_A of an algebra B, for which we refer to [5].

1. ν-orbits of Generic Modules of Repetitive Algebras. Let $A = k\vec{\Delta}$ be a tame hereditary algebra over an k. We denote by $D^b(A)$ the derived category $D^b(\mathrm{mod}\,A)$ of bounded complexes over $\mathrm{mod}\,A$. For the definition of derived category we refer to [7]. By DA we denote the minimal injective cogenerator of A, where $D = \mathrm{Hom}_k(-,k)$ is the usual dual functor. Consider the repetitive algebra [7]:

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