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EXISTENCE AND UNIQUENESS OF STANDARD BUBBLE CLUSTERS OF GIVEN VOLUMES IN $\mathbb{R}^N$ *

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Abstract. We prove the existence of a unique standard k-bubble in $\mathbb{R}^n$, $k \leq n+1$, that encloses fixed volumes.

1. Introduction. A conjecture of much recent interest [SM, Prob. 2] says that a standard soap bubble cluster consisting of spherical surfaces meeting in threes at 120 degrees provides the least-perimeter way to enclose and separate k regions of prescribed volume in $\mathbb{R}^n$ for $k \leq n+1$. Here we provide proofs of the existence and uniqueness of such a standard soap bubble.

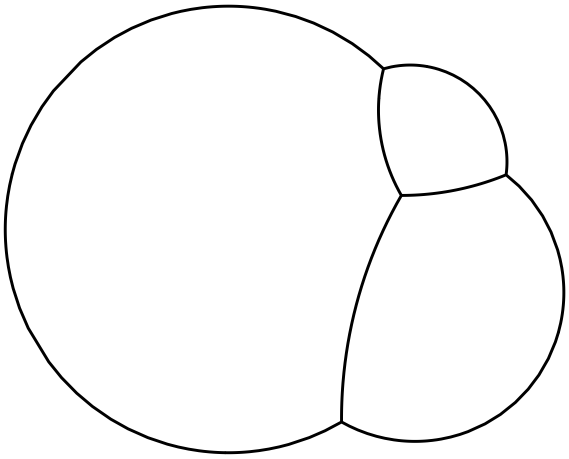

Fig. 1. *Standard 3-bubble in* $\mathbb{R}^2$

Progress on the conjecture. As for the conjecture itself, the case $k = 1$ is the classical theorem that the round sphere provides the least-perimeter way to enclose given volumes in $\mathbb{R}^n$. The case $k = 2$ is the Double Bubble Conjecture, proved in $\mathbb{R}^2$ in 1990 by a group of undergraduates [F2] and in $\mathbb{R}^3$ in 1995 for the case of equal volumes with the help of a computer by Hass, Hutchings, and Schlafly [HHS]. See [M1].

The case $k = 3$, the Triple Bubble Conjecture, was proved for connected regions in $\mathbb{R}^2$ in 1992 by another group of undergraduates (including Hutchings) [C].

General results on the existence and regularity of minimizers are due to Almgren and Taylor ([A], [T], [AT]), Morgan [M2], and Hutchings [H, 2.6, 2.9, 5.1].

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