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RETICULAR LEGENDRIAN SINGULARITIES*

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Dedicated to Professor Takuo Fukuda on his sixtieth birthday

1. Introduction. In [6] K. Jänich explained the wavefront propagation mechanism on a manifold which is completely described by a positive and positively homogeneous Hamiltonian function on the cotangent bundle and investigated the local gradient models given by the ray length function. Wavefronts generated by an initial wavefront which is a hypersurface without boundary in the manifold is investigated as Legendrian singularities by V.I.Arnold (cf., [1]). In [9], I.G.Scherbak studied the case when the hypersurface has a boundary and she explained the wavefronts generated by the hypersurface with a boundary corresponds to a generalized notion of wavefronts (i.e., the *boundary fronts*).

In this paper we investigate the more general case when the hypersurface has an *r-corner*. In this case each wavefront incident from each edge of the hypersurface gives a *contact regular r-cubic configuration* (cf., Section 5) at a point of the 1-jet bundle which is a generalization of the notion of Legendrian submanifolds. In complex analytic category, the notion of contact regular r-cubic configurations is introduced by Nguyen Huu Duc, Nguyen Tien Dai and F.Pham (cf., [2], [5]). But all contact regular r-cubic configuration in their category is stable.

The first topic in this paper is the investigation of the relation between (symplectic) regular r-cubic configurations which has been developed in [10] and contact regular r-cubic configurations.

The second topic is the investigation of the stability of smooth contact regular r-cubic configurations and the classification of *stable wavefonts* given by stable contact regular r-cubic configurations in C^∞-category. In order to realize this purpose we shall define the notion of *reticular Legendrian maps* in Section 7 which is a generalization of the notion of Legendrian maps for our situations. We shall also give the theorem that the equivalence relation among reticular Legendrian maps is equivalent to a certain equivalence relation of corresponding generating families. In this section we shall define the notion of *stability, homotopically stability, infinitesimal stability* of reticular Legendrian maps and give the theorem that these and the stability of corresponding generating families are all equivalent.

By the above results the classification of stable wavefronts is reduced to the classifications of function germs. In section 8 we classify function germs with respect to *reticular K-equivalence* with *reticular K-codimension* lower than 8 . This gives the classification of stable wavefonts in manifolds of dimension≤ 7.

Here, we draw the figure of the wavefront of one of the reticular Legendrian map-germ whose generating family is a reticular versal unfolding of $B^{-;-}_{2,3}$-singularity in the classification list, that is $x_1^2-x_1x_2-x_2^3+q_1x_2^2+q_2x_2+q_3x_1+q_4$. The wavefront given by this generating family is a subset in (q_1,q_2,q_3,q_4)-space around 0. Hence we draw the sections of this wavefront in (q_2,q_3,q_4)-space given by cutting at $q_1<0, q_1=0, q_1>0$ respectively.

2. Preliminaries. Here we shall define several notations and recall basic facts. Let $\mathbf{H}^r = \{(x_1,\cdots,x_r)\in\mathbf{R}^r | x_1\geq 0,\cdots,x_r\geq 0\}$ be an r-corner. Let $\mathcal{E}(r;l)$ be

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