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FINITE TYPE INVARIANTS OF KNOTS VIA THEIR SEIFERT MATRICES*

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Abstract. We define a filtration on the vector space spanned by Seifert matrices of knots related to Vassiliev's filtration on the space of knots. Further we show that the invariants of knots derived from the filtration can be expressed by coefficients of the Alexander polynomial.

The theory of finite type invariants (Vassiliev invariants) for knots was first introduced by V. Vassiliev [13] and reformulated by J. S. Birman and X. S. Lin [4]. M. Kontsevich defined the universal Vassiliev invariant [9, 1] by using iterated integral. The invariant takes values in the linear combinations of chord diagrams and one can use it to construct an isomorphism from the space of all the Vassiliev invariants of degree d to the chord diagrams with d chords modulo diagrams with more chords.

D. Bar-Natan [1] extended the notion of chord diagrams allowing trivalent vertices, which we call web diagrams in this paper. He showed that the space of (the linear combinations of) chord diagrams modulo the four-term relation coincides with the space of web diagrams modulo the AS, IHX and STU relations. So a main interest in the theory of Vassiliev invariants is the study of web diagrams.

In this paper we consider a classical knot invariant, the (S-equivalent classes of) Seifert matrices related to Vassiliev invariants. Let $\mathcal{K}$ be the vector space over $\mathbb{Q}$ spanned by knots and let

$$\mathcal{K} \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \mathcal{K}_3 \supset \cdots$$

be Vassiliev's filtration of $\mathcal{K}$. A Vassiliev invariant of degree d is defined to be a map $\mathcal{K}/\mathcal{K}_{d+1} \to \mathbb{Q}$. Further let $\mathcal{S}$ be the vector space spanned by Seifert matrices. There is a natural map $s : \mathcal{K} \to \mathcal{S}$ which takes a knot to its Seifert matrix. We consider a filtration

$$\mathcal{S} \supset \mathcal{S}_1 \supset \mathcal{S}_2 \supset \mathcal{S}_3 \supset \cdots$$

of $\mathcal{S}$ induced from Vassiliev's filtration by s. Our motivation is that which finite type invariant factors $\mathcal{S}/\mathcal{S}_{d+1}$. Since the Alexander-Conway polynomial can be defined by using Seifert matrices, the finite type invariants coming from the Alexander-Conway polynomial factor $\mathcal{S}/\mathcal{S}_{d+1}$. The main result of this paper is that these are all that factor it.

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