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HARMONIC RADIAL COMBINATIONS AND DUAL MIXED VOLUMES*

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Abstract. For star bodies, p-harmonic radial combinations were introduced and studied in several papers. In this paper we study the relations of the dual quermassintegrals of p-harmonic radial combinations of star bodies to their dual quermassintegrals and obtain the upper bound for the dual quermassintegrals of p-harmonic radial combinations of star bodies.

For star bodies, p-harmonic radial combinations were introduced and studied in several papers. The aim of this article is to study them further, that is, we investigate the relations of the dual quermassintegrals of p-harmonic radial combinations of star bodies to their dual quermassintegrals and obtain the upper bound for the dual quermassintegrals of p-harmonic radial combinations of star bodies.

1. Preliminaries. By a *convex body* in $E^n, n \geq 2$, we mean a compact convex subset of E^n with nonempty interior. Let S^{n-1} denote the unit sphere centered at the origin in E^n, and write O_{n-1} for the $(n-1)$-dimensional volume of S^{n-1}. Let B be the closed unit ball in E^n, write ω_n for the n-dimensional volume of B. Note that

$$\omega_n = \pi^{n/2}/\Gamma(1+\frac{1}{n}), \qquad \text{and} \qquad O_{n-1} = n\omega_n.$$

For each direction $u \in S^{n-1}$, we define the support function $h(K,u)$ on S^{n-1} of the convex body K by

$$h(K,u) = \sup\{u \cdot x | x \in K\}$$

and the radial function $\rho(K,u)$ on S^{n-1} of the convex body K is

$$\rho(K,u) = \sup\{\lambda > 0 | \lambda u \in K\}.$$

If $\rho(K,u)$ is positive and continuous, call K a *star body* (about the origin), and write $\mathcal{S}$ for the set of star bodies (about the origin) of E^n. Sets A, B are called *homothetic* if $A = \lambda B + t$ with $t \in E^n$ and $\lambda > 0$ or one of them is a singleton (a one-point set).

The *polar body* of a convex body K, denoted by K^*, is another convex body defined by

$$K^* = \{y | x \cdot y \ \leq 1 \text{ for all } \ x \in K\}.$$

The polar body has the well known property that

$$h(K^*,u) = 1/\rho(K,u) \ \text{ and } \ \rho(K^*,u) = 1/h(K,u),$$

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