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FINITE JET DETERMINATION OF HOLOMORPHIC MAPPINGS AT THE BOUNDARY*

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0. Introduction. A classical theorem of H. Cartan ([HCa]) states that an automorphism f of a bounded domain $D \subset \mathbb{C}^N$ is completely determined by its 1-jet, i.e. its value and derivatives of order one, at any point $Z_0 \in D$. If D, in addition, is assumed to be smoothly (C^∞) bounded and strictly pseudoconvex, then by Fefferman's theorem [Fe] any such automorphism extends smoothly to the boundary ∂D as an automorphism $\partial D \to \partial D$. It is then natural to ask: is f completely determined by a finite jet at a boundary point $p \in \partial D$? An affirmative answer to this question, when D is strictly pseudoconvex, follows from the work of Chern and Moser [CM] (see also E. Cartan [ECa1–2] for the case $N = 2$, and Tanaka [T1–2]). Indeed, the following local version of Cartan's theorem is a consequence of their work. *Any holomorphic mapping which is defined locally on one side of a smooth, Levi nondegenerate real hypersurface $M \subset \mathbb{C}^N$ and extends smoothly to M, sending M diffeomorphically into another smooth real hypersurface $M' \subset \mathbb{C}^N$, is completely determined by its 2-jet at a point $p \in M$.* Observe that the conclusion is nontrivial even in the strictly pseudoconcave case when the mapping extends holomorphically to a full neighborhood of p.

The main objective of the present paper is to extend the above mentioned local result to a more general class of real hypersurfaces (Theorem 1 below). We should point out that the result for Levi nondegenerate hypersurfaces follows from the construction of a unique Cartan connection on a certain principal G-bundle over such a hypersurface. There is no analogue of this construction in the more general situation considered in this paper.

Let $M \subset \mathbb{C}^N$ be a smooth real hypersurface and assume that M is defined locally near a point $p_0 \in M$ by the equation $\rho(z, \bar z) = 0$, where ρ is a smooth function with $\rho(p_0, \bar p_0) = 0$ and $d\rho(p_0, \bar p_0) \neq 0$. Let $L_{\bar 1}, \ldots, L_{\bar n}$, with $n = N - 1$, be a basis for the CR vector fields on M. We shall say that M is *k_0-nondegenerate* at p_0 if

$$\text{Span}\left\{L^{\bar\alpha}\rho_z(p_0, \bar p_0) \colon |\alpha| \le k_0\right\} = \mathbb{C}^N, \tag{0.1}$$

where $\rho_z := (\partial_{z_j}\rho)_{1\le j\le N}$, $\partial_{z_j} := \partial/\partial z_j$, and standard multi-index notation for differential operators is used i.e. $L^{\bar\alpha} := L_{\bar 1}^{\bar\alpha_1} \ldots L_{\bar n}^{\bar\alpha_n}$. This nondegeneracy condition will be given in a different, but equivalent, form in terms of the intrinsic geometry of M in the next section. The reader is referred to the book [BER3] for basic material on real submanifolds in complex space and CR structures, and further discussion of various nondegeneracy conditions (see also §1 of the present paper). We mention here only that Levi nondegeneracy at a point $p_0 \in M$ is equivalent to 1-nondegeneracy. Our main result is the following.

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