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A CLASSIFICATION OF NON-DEGENERATE HOMOGENEOUS EQUIAFFINE HYPERSURFACES IN FOUR COMPLEX DIMENSIONS*

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Abstract. We solve the classification problem as in the title. We present explicit defining equations and give a characterization in terms of easily computable affine invariants.

1. Introduction. The *equiaffine* transformations of $\mathbb{C}^4$ are those of the form

$$v \longmapsto Av + b \quad \text{for } A \in \mathrm{SL}(4, \mathbb{C}) \text{ and } b \in \mathbb{C}^4.$$

A complex hypersurface $\Sigma \subset \mathbb{C}^4$ is said to be *equiaffine homogeneous* if and only if it is the orbit of a complex Lie subgroup of the group of all equiaffine transformations. Such a hypersurface is everywhere non-singular. We shall also suppose that Σ is *non-degenerate* (at one point and hence, by homogeneity, at all points). In this article, we classify the non-degenerate equiaffine homogeneous hypersurfaces in $\mathbb{C}^4$ up to affine equivalence.

The corresponding classification problem for surfaces in $\mathbb{C}^3$ was solved by Nomizu and Sasaki [9]. There are six such surfaces:–

1. $Z = XY$
2. $Z^2 = XY + 1$
3. $XYZ = 1$
4. $X^2(Z + Y^2)^3 = 1$
5. $Z = XY + X^3$
6. $Z = XY + \log X$

The first five were found by Guggenheimer [5] and this list was completed in [9]. See [10] for further discussion and for general background on affine differential geometry.

In [1], the affine homogeneous surfaces in $\mathbb{C}^3$ were classified by means of normalizing their defining functions. The equiaffine cases have

1. $z = xy + \mathrm{O}(5)$
2. $z = xy + x^2y^2 + \mathrm{O}(5)$
3. $z = xy + x^3 + y^3 + \mathrm{O}(5)$
4. $z = xy + x^3 + y^3 + \frac{9}{8}x^4 - \frac{9}{2}x^3y - \frac{9}{4}x^2y^2 - \frac{9}{2}xy^3 + \frac{9}{8}y^4 + \mathrm{O}(5)$
5. $z = xy + x^3 + \mathrm{O}(5)$
6. $z = xy + x^3 + x^4 + \frac{16}{15}x^5 + \mathrm{O}(6)$

as their normal forms. The higher order terms are determined by the given terms together with the requirement that the surface be affine homogeneous. In [1] this list is derived by choosing affine coördinates specially adapted to the surface and

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