ASIAN J. MATH.
Vol. 6, No. 1, pp. 169–194, March 2002

009

TWISTED TORSION ON COMPACT HYPERBOLIC SPACES: A REPRESENTATION-THEORETIC APPROACH*

MARIA G. GARGOVA FUNG†

Abstract. In this paper we consider a twisted version τ_θ of the Ray-Singer analytic torsion on compact locally symmetric spaces $X = K\backslash G/\Gamma$ (with G a noncompact connected semisimple Lie group, K its maximal compact subgroup, and Γ a discrete torsion-free cocompact subgroup), where θ is an automorphism of X with the property that $\theta^2 = 1$. We obtain a representation-theoretic interpretation of the twisted torsion via certain irreducible unitary representations of G. By considering θ = Cartan involution for $SO_0(2n+1,1)$, we show that $|\tau_\theta| = 1$ for the compact hyperbolic spaces associated to this family of Lie groups.

1. Introduction. The analytic torsion of a compact Riemannian manifold, introduced first by Ray and Singer in [18], is an important invariant that allows us to distinguish between spaces with isomorphic cohomology rings and homotopy groups. It is composed of the spectral information of the Laplacian operators associated to the De Rham complex of the manifold. Cheeger and Müller independently (see [5] and [17]) have shown that this torsion coincides with its combinatorial counterpart, the Reidemeister torsion.

In the special case when the manifold is a compact locally symmetric space $X = K\backslash G/\Gamma$, with G a real semisimple non-compact Lie group, K its maximal compact subgroup and Γ a torsion-free cocompact discrete subgroup, Speh in [22] expresses the analytic torsion in terms of representation-theoretic data. Inspired by work of Fried [7] on compact hyperbolic spaces, she describes the spectrum of the Laplacian operators via certain irreducible representations in the unitary dual of G. This allows Speh to construct a proof different from that of Moscovici and Stanton [16] of the vanishing of the torsion on all compact locally symmetric spaces of the type above, except in the cases when G has a factor locally isomorphic to $SO(p,q)$ with pq odd, or $SL(3,\mathbb{R})$.

An automorphism of a compact Riemannian manifold acts on the Laplacian operators associated to its complex of differential forms, and thus it acts on the building blocks of the analytic torsion. Therefore, each such automorphism allows us to construct a twisted invariant analogous to the usual torsion. In this paper we do this in the case when θ is an involution of a compact locally symmetric space. We prove the following vanishing result:

THEOREM 4.2 *For the compact locally symmetric space $X = K\backslash G/\Gamma$ with G locally isomorphic to $SO_0(2n+1,1)$, K its maximal compact subgroup and Γ a discrete torsion-free subgroup, the twisted torsion*

$$|\tau_\theta| = 1$$

for all outer automorphisms of G such that $\theta^2 = 1$ and $\theta(K) = K$, $\theta(\Gamma) = \Gamma$.

The proof of this theorem is modeled after Speh's proof of the vanishing of the analytic torsion. Thus, we first obtain a representation-theoretic interpretation of the twisted torsion on a general compact locally symmetric space in 2.3. Then we relate this invariant of the locally symmetric space to "twisted torsion" of θ-invariant unitary

*Received August 5, 2001; accepted for publication November 16, 2001.

†Department of Mathematics, Western Oregon Univeristy, Monmouth, OR 97361, USA. The author would like to extend special gratitude to Birgit Speh for all her direction, encouragement, and patience during the completion of this doctoral research at Cornell University.