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PSEUDO-HAMILTONIAN REALIZATION AND ITS APPLICATION*

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Abstract. In this paper, the problem of pseudo-Hamiltonian realization of a control system is studied. Several sufficient conditions are obtained. The stability of a dynamic system is investigated via dissipative pseudo-Hamiltonian realization, and the stabilization of a control system is also investigated via feedback dissipative pseudo-Hamiltonian realization. Some relations between the stability (asymptotical stability) with the dissipative (strict dissipative) realization are revealed. Relations between the affine dissipative control systems and the dissipative pseudo-Hamiltonian realization are also investigated. These results show that the set of pseudo-Hamiltonian systems represent a very large class of interesting dynamic systems, and this approach is a powerful tool. Particularly, a generalization of the Krasovskii's Theorem is obtained. Then the problem of L_2 disturbance attenuation of a nonlinear system via pseudo-Hamiltonian realization is investigated. It is shown that for a class of pseudo-Hamiltonian systems the disturbance attenuation problem is solvable and an estimation of the boundary of the L_2 gain is obtained. Finally the results are applied to the excitation control of power systems. The stabilization and the H_∞ control problems are investigated for the single-machine infinite bus power systems.

Keywords. Hamiltonian system, dissipative system, stabilization, disturbance attenuation, power system

1. Introduction. In recent years, the problem of energy-based Lyapunov functions was investigated intensively. The theory of the passivity-based control has been well-established [16],[18],[25]. Particularly, the port-controlled Hamiltonian systems were studied by [11], [15], [17]. It becomes a powerful technique for designing robust controllers for many physical systems, which are described as a "generalized" Hamiltonian systems. Some applications of the approach were illustrated in [5], [7], [20], [22],[26], [27].

The advantage of this approach is that for this class of systems when the stability related problems are investigated, the Lyapunov candidates can be chosen from the Hamiltonian functions. When the robust or H_∞ control problems are considered, the Hamiltonian functions may serve as the storage function, to avoid solving HJI inequality, etc.

A pseudo-Hamiltonian dynamic system was proposed in [4], [8] as

$$\dot{x} = M(x)\frac{\partial H}{\partial x}, \quad x \in \mathbb{R}^n, \tag{1}$$

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where $M(x)$ is an $n \times n$ matrix with entries as C^r function on $\mathbb{R}^n \backslash \{0\}$, called the *structure matrix*. $H \in C^r(\mathbb{R}^n)$ is the Hamiltonian function of the system. We also denote by $X_H = M(x)\frac{\partial H}{\partial x}$, the Hamiltonian vector field deduced by H.

Through the paper we use C^r for certain $r > 0$ to assure sufficiently many differentiability.

A controlled pseudo-Hamiltonian system is defined as

$$\begin{cases} \dot{x} = M(x)\frac{\partial H}{\partial x} + \sum_{i=1}^{m} g_i u_i, \quad x \in \mathbb{R}^n \\ y = g^T \frac{\partial H}{\partial x}, \end{cases} \tag{2}$$

where $M(x)$ and $H(x)$ are as in (1), $g_i(x)$, $i = 1, \cdots, m$, are C^r vector fields, and $g = [g_1 \ g_2 \ \cdots g_m]$.

When a local problem is discussed, $\mathbb{R}^n$ is replaced by an open neighborhood, U of the origin, i.e., $0 \in U \subset \mathbb{R}^n$.

As proposed in [8], we allow $M(x)$ to be an arbitrary matrix. Decompose $M(x) = K(x) + P(x)$, where $K(x)$ is skew-symmetric and $P(x)$ is symmetric. Furthermore, assume x is a regular point of $P(x)$ in the sense that the number of positive eigenvalues and the number of negative eigenvalues are locally invariant. Then we may further decompose $P(x) = -R(x)+T(x)$, where both $R(x)$ and $T(x)$ are positive semi-definite and the ranks of $T(x)$ and $R(x)$ are the numbers of positive eigenvalues and negative eigenvalues respectively. Then under the regularity assumption, we have a unique decomposition of $M(x)$ as

$$M(x) = K(x) - R(x) + T(x). \tag{3}$$

Similar decomposition may also be found in [14].

In later discussion, we assume $K(x)$, $R(x)$, and $T(x)$ are C^r on $\mathbb{R}^n \backslash \{0\}$.

We call system (1) a dissipative pseudo-Hamiltonian system, system (2) a controlled dissipative pseudo-Hamiltonian system or port-controlled Hamiltonian system if $T(x) \equiv 0$. Port-controlled Hamiltonian system was proposed and studied by Ortega, Van der Shaft et al. [11], [15], [16], [17], [18], [19], [20], [25].

The generalization, provided by the concept of *pseudo-Hamiltonian system* is to allow $T(x) \neq 0$. The motivation for this generalization lies on the following two points: First of all, converting an affine nonlinear system into a port-controlled Hamiltonian system directly is difficult. But from the later discussion one sees easily that, roughly speaking, almost all the affine nonlinear systems can be converted to the pseudo-Hamiltonian systems. Moreover, almost all the functions can be the Hamiltonian function for a given nonlinear system. So this approach can cover a very large class of systems. Secondly, some conditions are known to convert a pseudo-Hamiltonian system to the port-controlled system via feedback [6]. This problem will be further studied in this paper. Then a two step realization can be proposed as: dynamic system $\rightarrow$ pseudo-Hamiltonian system $\rightarrow$ port-controlled system, and the well established theory for port-controlled systems may be used for a large class of control systems.

Even though the pseudo-Hamiltonian systems are so general, they still have many interesting properties. We refer to [8] for some studies, which may be convincing for exploring this kind of systems.

The energy-based Lyapunov function approach has been used in the control of power systems [5],[22], [26], [27]. A key point in applying this new approach to a general control system is to express the system as a controlled pseudo-Hamiltonian system and further to a port-controlled Hamiltonian model. In [15] a constructive methodology was presented to design controllers for a class of systems of the form of port-controlled Hamiltonian system preserving such a structure. The standard Hamiltonian realization problem has been studied widely in eighties of the last century. We refer to [10] and the references therein for the realization of classical (controlled) Hamiltonian systems. But to the authors' knowledge, there is no systematic method to handle the problem of (dissipative) pseudo-Hamiltonian realization for general (control) systems.

The first purpose of this paper is to explore a possible solution to the pseudo-Hamiltonian realization and the dissipative pseudo-Hamiltonian realization.

Then the stability via dissipative realization and the stabilization via feedback dissipative realization are investigated. It is shown that under certain conditions, a port-controlled system is equivalent to a dissipative pseudo-Hamiltonian system. Moreover, a stable (asymptotically stable) system is equivalent to a dissipative (strict dissipative) pseudo-Hamiltonian system with a positive definite Hamiltonian function. In particular, a generalized Krasovskii's Theorem is presented.

Using the pseudo-Hamiltonian form, it is shown that for a class of pseudo-Hamiltonian systems the disturbance attenuation problem is solvable. Moreover, an estimation of the boundary of the L_2 gain is obtained.

Finally, the results are applied to the excitation control of power systems. It is shown that for a single machine system the dissipative pseudo-Hamiltonian realization is unique and can be obtained mechanically. The H_∞ control is obtained under more general disturbance than [21]. Using a set of engineering parameters the precise boundary of the L_2 gain is calculated.

The paper is organized as follows: In section 2, the pseudo-Hamiltonian realization is considered. Section 3 considers the problem of dissipative pseudo-Hamiltonian realization and its applications to stability problems. The stabilization problem is considered in section 4 via feedback dissipative pseudo-Hamiltonian realization. In section 5 we consider the disturbance attenuation problem of a port-controlled Hamiltonian system. Finally, in section 6 the excitation control of power systems is studied by pseudo-Hamiltonian approach. Section 7 is the conclusion.

2. Pseudo-Hamiltonian Realization.

Definition 2.1. *Consider a dynamic system*

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n, \tag{4}$$

where $f(x) = [f_1(x)\ f_2(x) \cdots f_n(x)]^T$ *is* C^r *vector field with* $f(0) = 0$. *The system (4) is said to have a pseudo-Hamiltonian realization if there exists a suitable coordinate charge and a* C^r *function* H, *such that equation (4) can be converted into a pseudo-Hamiltonian system (1).* $H(x)$ *is then called the Hamiltonian function of the system.*

Consider a controlled dynamic system

$$\begin{cases} \dot{x} = f(x) + \sum\limits_{i=1}^{m} g_i(x)u_i, \quad x \in \mathbb{R}^n \\ y = h(x), \quad y \in \mathbb{R}^m, \end{cases} \tag{5}$$

where $f(x)$, $g_i(x)$, $i = 1, \cdots, m$ *are* C^r *vector fields,* $h(x)$ *is a set of* m C^r *functions, and* $f(0) = 0$, $h(0) = 0$. *The system (5) is said to have a pseudo-Hamiltonian realization if there exists a suitable coordinate change and a* C^r *function* H *,called the Hamiltonian function, such that the equation (5) can be converted into a controlled pseudo-Hamiltonian system (2).*

If in a realization as the form of (1) ((2)), the decomposition of $M(x)$ *has* $T(x) \equiv 0$, *it is called a dissipative pseudo-Hamiltonian realization (or controlled dissipative pseudo-Hamiltonian realization respectively). If in addition,* $R(x)$ *is positive definite, the (controlled) dissipative realization is called a (controlled) strict dissipative realization.*

To begin with, we may propose a Hamiltonian function, $H(x)$, in advance and try to convert a dynamic system into a pseudo-Hamiltonian system with $H(x)$ as its Hamiltonian function. For convenience, through the paper, for a smooth function $H(x)$ we denote

$$dH(x) = (\frac{\partial H(x)}{\partial x_1}, \cdots, \frac{\partial H(x)}{\partial x_n}) = \left(\frac{\partial H(x)}{\partial x}\right)^T.$$

PROPOSITION 2.2. *Let* $H(x)$ *be a* C^r *function. If* $\|dH(x)\| := \sqrt{dH(x)\frac{\partial H}{\partial x}(x)} \neq 0$, $x \neq 0$, *then the system (4) has a pseudo-Hamiltonian realization with* $H(x)$ *as its Hamiltonian function.*

Proof. Choosing the structure matrix, $M(x) = (m_{ij}(x))$, as

$$m_{ij}(x) = \begin{cases} \frac{f_i(x)dH(x)_j}{\|dH(x)\|^2}, \quad x \neq 0, \\ 0, \quad x = 0. \end{cases} \tag{6}$$

Then $M(x)$ is smooth on $\mathbb{R}^n \backslash \{0\}$ and for this $M(x)$ system (4) becomes (1). □

REMARK 2.3. *It is obvious that the structure matrix is not unique. Say, for any smooth function* $\psi(x)$ *with* $\psi(0) = 0$ *if we replace* $m_{ij}(x)$ *and* $m_{ik}(x)$ *simultaneously by*

$$m_{ij} - \psi(x), \qquad m_{ik}(x) + \frac{\psi(x)dH(x)_j}{dH(x)_k}, \quad x \neq 0 \tag{7}$$

respectively, $M(x)$ remains available. In this way we can modify $M(x)$ to meet our further requirement. For instance, see the following example.

EXAMPLE 2.4. Consider a system

$$\dot{x} = \begin{pmatrix} -x_1 \\ -x_1^2 x_2 \end{pmatrix}. \tag{8}$$

Let $H(x) = \frac{1}{2}(x_1^2 + x_2^2)$. Using (6) directly, we have

$$\dot{x} = \begin{pmatrix} -\frac{x_1^2}{x_1^2+x_2^2} & -\frac{x_1 x_2}{x_1^2+x_2^2} \\ -\frac{x_1^3 x_2}{x_1^2+x_2^2} & -\frac{x_1^2 x_2^2}{x_1^2+x_2^2} \end{pmatrix} \frac{\partial H(x)}{\partial x}. \tag{9}$$

For $i = 1$, $j = 1$, $k = 2$, choose $\psi_1 = \frac{x_2^2}{x_1^2+x_2^2}$, and for $i = 2$, $j = 1$, $k = 2$, choose $\psi_2 = \frac{x_1 x_2^3}{x_1^2+x_2^2}$, then (7) converts (9) to

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ -x_1 x_2 & 0 \end{pmatrix} \frac{\partial H(x)}{\partial x}, \tag{10}$$

which is smooth at the origin. If in the above process, ϕ_2 is replaced by $\psi_2 = \frac{-x_1^3 x_2}{x_1^2+x_2^2}$, a dissipative realization is obtained as

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -x_1^2 \end{pmatrix} \frac{\partial H(x)}{\partial x}. \tag{11}$$

□

We give some simple examples to describe the realization.

EXAMPLE 2.5. (i). A simple form of $H(x)$ is the variable-separated function as

$$H(x) = \sum_{j=1}^{n} \psi_j(x_j), \tag{12}$$

where $\psi_j(x_j)$ are n smooth functions (C^r) and $\psi_j(0) = 0$.

Then the system (4) has a pseudo-Hamiltonian realization if there exist continuous functions $\phi_j(x_j)$, $m_{ij}(x)$, $i = 1, \cdots, n$; $j = 1, \cdots, n$, such that

$$f_i(x) = \sum_{j=1}^{n} m_{ij}(x)\phi_j(x_j), \quad i = 1, \cdots, n, \tag{13}$$

because we can simply choose $\psi_j(x_j) = \int_0^{x_j} \phi_j(\tau)d\tau, \quad j = 1, \cdots, n$. Then (4) becomes

$$\dot{x} = M(x)\frac{\partial H}{\partial x},$$

where $M(x) = (m_{ij}(x))$ and $H(x)$ is as in (12).

(ii). To illustrate the model described in (i), consider a particle in a gravity field. Let x be the displacement and $v = \dot{x}$ be the velocity. According to the Newton's second law we have

$$\begin{cases} \dot{x} = v \\ \dot{v} = -c\frac{1}{(x+R_0)^2}, \quad x > 0, \end{cases} \tag{14}$$

where R_0 is the radius of the other body, say earth. According to (12), we may simply choose $\phi_1 = -c\frac{1}{x^2}$ and $\phi_2 = mv := p$, which is the momentum of the particle, then we get the Hamiltonian function as

$$H = \frac{1}{2m}p^2 + c\left(\frac{1}{R_0} - \frac{1}{R_0 + x}\right),$$

and (14) becomes a standard Hamiltonian system:

$$\begin{pmatrix} \dot{x} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & m \\ -m & 0 \end{pmatrix} \frac{\partial H}{\partial(x,p)}. \tag{15}$$

(iii) Consider the system (4), assume the Jacobian matrix, $J_f(x)$, of f is symmetric. Then as shown in [12] (4) can be expressed as $\dot{x} = (-I)\frac{\partial H}{\partial x}$ with the Hamiltonian function as

$$\begin{array}{rcl} H(x) & = & -\int_0^{x_n} f_n(x_1, \cdots, x_{n-1}, \tau_n)d\tau_n \\ & & -\int_0^{x_{n-1}} f_{n-1}(x_1, \cdots, x_{n-2}, \tau_{n-1}, 0)d\tau_{n-1} \cdots \\ & & -\int_0^{x_1} f_1(\tau_1, 0, \cdots, 0)d\tau_1. \end{array}$$

□

To assure the positivity of the Hamiltonian function we assume $\psi_i(x_i) > 0$ and $\psi_i(0) = 0$, which make $H(x)$ be a candidate of a Lyapunov function. Particularly, we may assume $\psi_i(x_i) = x_i^2, \quad i = 1, \cdots, n$. Then we have the following proposition, which is basically well known [13], but with a slightly different statement.

Proposition 2.6. *System (4) has a pseudo-Hamiltonian realization with*

$$H(x) = \frac{1}{2}\sum_{i=1}^{n} x_i^2,$$

iff $f(0) = 0$.

Proof. Necessity is obvious. As for sufficiency, if $f(0) = 0$ system (4) can be converted to (1) with the structure matrix as

$$M(x) = \int_0^1 \left[\frac{\partial f_1(\tau)}{\partial \tau}, \frac{\partial f_2(\tau)}{\partial \tau}, \cdots, \frac{\partial f_n(\tau)}{\partial \tau}\right]^T \Bigg|_{\tau = tx} dt.$$

□

The above discussion in this section shown that a large class of dynamic systems can be converted into the pseudo-Hamiltonian systems. Then the developed and developing theory on "generalized" Hamiltonian systems may be used to solve various control problems. Hence it is worthwhile to investigate such class of systems.

Consider system (4) again. A practically useful case is to convert it into (1) with constant M. A constant M provides a Lie-group and Lie-algebra structure for the system, which are parallel to the symplectic group and symplectic algebra [8].

Define a set of row vectors $A_i = (\frac{\partial}{\partial x_i} f)^T, \quad i = 1, \cdots, n$. We have the following:

PROPOSITION 2.7. *System (4) has a constant and invertible M realization if the following equation*

$$\begin{pmatrix} A_2 & -A_1 & & & \\ A_3 & & -A_1 & & \\ & & & \ddots & \\ A_n & & & & -A_1 \\ & A_3 & -A_2 & & \\ & & & \ddots & \\ & A_n & & & -A_2 \\ & & \vdots & & \\ & & & A_n & -A_{n-1} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} = 0, \quad , X_i \in \mathbb{R}^n, \ i = 1, \cdots, n \tag{16}$$

has a constant solution $\{X_i\}$, which makes the following matrix N non-singular:

$$N = col(X_1^T, \ X_2^T \ \cdots \ X_n^T).$$

To see the pattern of the coefficient matrix in (16), note that it consists of $n-1$ blocks with $n-1$, $n-2$, $\cdots$, 1 rows in sequence.

Proof. Denote the i-th component of Nf by H_i. Then a straightforward computation shows that

$$\frac{\partial H_i}{\partial x_j} = A_j X_i, \quad i, j = 1, \cdots, n.$$

The solution of (16) implies that

$$\frac{\partial H_i}{\partial x_j} = \frac{\partial H_j}{\partial x_i}, \quad i \neq j.$$

According to Poincare's Lemma [1], (Nf) is closed. That is, there exists H, such that $(Nf) = \frac{\partial H}{\partial x}$. Hence, $f = M \frac{\partial H}{\partial x}$. Note that since $\mathbb{R}^n$ is simply connected a global solution of (16) provides a global realization. □

REMARK 2.8. *It is easy to prove that if the coordinate frame is fixed, the conditions in Proposition 2.7 are also necessary.*

3. Stability via Dissipative Realization. In this section we consider the stability of a dynamic system via pseudo-Hamiltonian realization. The following proposition follows from definition and a straightforward computation.

Proposition 3.1. *1. System (1) is a (strict) dissipative pseudo-Hamiltonian system iff*

$$M(x) + M^T(x) \leq 0 \quad (< 0). \tag{17}$$

2. Let $z = z(x)$ be a coordinate change with the Jacobian matrix J_z. Then the structure matrix under the new coordinate frame is

$$M(z) = J_z(x)M(x)J_z^T(x)|_{x=x(z)}. \tag{18}$$

where $x = x(z)$ denotes the inverse mapping of $z = z(x)$.

It follows from (18) that the decomposition in (3) is coordinate independent. Therefore, the dissipativity of a system is also coordinate independent.

Next, we investigate the relationship between the (asymptotical) stability with the (strict) pseudo-Hamiltoniant realization of a dynamic system. We start with linear systems.

Proposition 3.2. *A linear system*

$$\dot{x} = Ax \tag{19}$$

has a dissipative realization (strict dissipative realization) with positive Hamiltonian function, iff it is stable (asymptotically stable).

Proof. (Sufficiency) Case 1. A is asymptotically stable. The dissipative realization can be obtained in the following way: We can choose the Hamiltonian function as $H = \frac{1}{2}x^T Px$ with $P > 0$. Then the generalized Hamiltonian realization is

$$\dot{x} = AP^{-1}\frac{\partial H}{\partial x} = (\frac{1}{2}(AP^{-1} - P^{-1}A^T) + \frac{1}{2}(AP^{-1} + P^{-1}A^T))\frac{\partial H}{\partial x} := (K - R)\frac{\partial H}{\partial x}.$$

Now K is obviously skew symmetric, so if we can find a positive definite P such that $R > 0$, we are done. It is well known that for any given negative definite matrix $Q < 0$ the Lyapunov equation

$$AP^{-1} + P^{-1}A^T = Q := -2R < 0$$

has a positive definite solution $P > 0$, which is what we are looking for.

Case 2. A is stable. We may convert A into a real Jordan form. Then the system is decoupled into several subsystems, each subsystem contains a Jordan block. If the block is of the eigenvalues with negative real part, it becomes case 1. If the block is of the eigenvalues with zero real part, the eigenvalue should have fold 1 because A is stable. So it is either 1×1 block of zero, as $J = (0)$, or 2×2 block as $J = \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$. Simply choosing $P = I$, it works.

(Necessity) If a linear system has a dissipative realization, then the Hamiltonian function should be a quadratic form. In addition, since the Hamiltonian function is required to be positive definite it should be $H = \frac{1}{2}x^T Px$, with $P > 0$. Using H as the Lyapunov function, since the structure matrix is (strict) dissipative the linear system is obviously (asymptotically) stable. □

Now a natural question is: for a nonlinear system whether the (strict) dissipativity with positive Hamiltonian function is also equivalent to (asymptotical) stability? To answer this and for the further investigation we propose some notations and concepts.

Let $M(x) = (m_{ij}(x))$ be a $p\times q$ matrix. Then $DM(x)$ is a $p\times(nq)$ matrix, obtained by replacing $m_{ij}(x)$ by its differential $dm_{ij}(x) = (\frac{\partial m_{ij}(x)}{\partial x_1}, \cdots, \frac{\partial m_{ij}(x)}{\partial x_n})$. The higher differentials can be defined inductively as

$$D^k M(x) = D(D^{k-1}M(x)), \quad k > 1.$$

Similarly, $\frac{\partial M(x)}{\partial x}$ is a $(np) \times (q)$ matrix, obtained by replacing $m_{ij}(x)$ by its gradient

$$\frac{\partial m_{ij}}{\partial x}(x) = (\frac{\partial m_{ij}(x)}{\partial x_1}, \cdots, \frac{\partial m_{ij}(x)}{\partial x_n})^T.$$

Moreover,

$$\frac{\partial^k}{\partial x^k}M(x) = \frac{\partial}{\partial x}(\frac{\partial^{k-1}}{\partial x^{k-1}}M(x)), \quad k > 1.$$

Let M and N be two matrices of dimensions $m \times n$ and $p \times q$ respectively. If $n = tp$ or $nt = p$ for some integer t, we define the *semi-tensor product* of M and N as

$$M \ltimes N = \begin{cases} M(N \otimes I_t), & n = tp \\ (M \otimes I_t)N, & nt = p. \end{cases} \tag{20}$$

Since it is a generalization of the conventional matrix product, we may omit $\ltimes$. Semi-tensor product is also associative [6], [9].

When the origin is an asymptotically stable equilibrium of the system (4) it must be an isolated equilibrium point. When the origin is globally asymptotically stable it must be the unique equilibrium point. So in the later discussion when the problem of asymptotical stability or global asymptotical stability is considered, the corresponding necessary condition is assumed. That means, $f(x) = 0$ implies $x = 0$ in either local or global case. We write it as a hypothesis.

H1. *The origin is an isolated equilibrium point of $f(x)$ when the local stability or stabilization problem is considered, it is the only equilibrium point when the global stability or stabilization problem is considered.*

As a consequence of **H1**, we have

PROPOSITION 3.3. *Assume H1 and the system (4) has a (strict) dissipative realization with a positive Hamiltonian function. Then the system is (asymptotically)* *stable. Moreover, if $H(x)$ is radially unbounded, the system is globally (asymptotically) stable.*

Proof. Taking H as the Lyapunov function, the stability follows. As for the asymptotical stability. Note that $\dot{H}(x) < 0$ and continuous for $x \neq 0$. Then for any given $\epsilon > 0$ there exists $T > 0$ such that $\|x(t)\| < \epsilon$ for $t > T$. In fact, it doesn't matter if $M(x)$ is discontinuous at zero. □

Now we are ready to consider the nonlinear counterpart of the Proposition 3.2.

Assume the system (4) is (globally) asymptotically stable with a C^r ($r \geq 2$) Lyapunov function L. Since the origin is a critical point, that is: $L(0) = 0$ and $\frac{\partial L}{\partial x}(0) = 0$, we can express $\frac{\partial L}{\partial x}$ as

$$\frac{\partial L}{\partial x} := \Phi(x)x. \tag{21}$$

According to Proposition 2.6, the system (4) can be expressed as

$$\dot{x} = A(x)x.$$

Now since the system (4) is (globally) asymptotically stable

$$\dot{L} = x^t \Phi^T(x) A(x) x < 0, \tag{22}$$

which holds (globally on $\mathbb{R}^n$) locally around the origin. A sufficient condition for 22 is

$$\Phi^T(x)A(x) + A^T(x)\Phi(x) < 0. \tag{23}$$

Now in addition to Proposition 3.3, we have the following

Proposition 3.4. *Assume the system (4) is (globally) asymptotically stable with a C^r ($r \geq 2$) Lyapunov function, $L(x)$ as in (21) with invertible $\Phi(x)$, and (23) holds. Then it has a (global) dissipative realization.*

Proof. Under the assumption it is easy to see that the system (4) can be expressed as

$$\dot{x} = M(x)\frac{\partial L(x)}{\partial x}, \tag{24}$$

where

$$M(x) = \begin{cases} A(x)\Phi^{-1}(x), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Since $\Phi^T(x)A(x) + A^T(x)\Phi(x)$ is a negative definite matrix, $x \neq 0$, we have that

$$A(x)\Phi^{-1}(x) + \Phi^{-T}(x)A^T(x) = \Phi^{-T}(x)(\Phi^T(x)A(x) + A^T(x)\Phi(x))\Phi^{-1}(x) < 0, \quad x \neq 0.$$

That is, the realization is a strict dissipative realization. □

A natural way to define a positive definite Hamiltonian function using f is to set

$$H(x) = f^t P(x) f, \quad \text{where } P(x) > 0. \tag{25}$$

A straightforward computation shows that

$$\frac{\partial H}{\partial x}(x) = [P(x)J_f(x) + J_f^T(x)P(x) + f^T(x) \ltimes \frac{\partial P(x)}{\partial x}]f.$$

Hence, we have the following realization:

LEMMA 3.5. *Assume there exists a matrix $P(x) > 0$, such that for the system (4) the matrix*

$$\Phi(x) := P(x)J_f(x) + J_f^T(x)P(x) + f^T(x) \ltimes \frac{\partial P(x)}{\partial x}$$

is nonsingular, then the system (4) has a realization

$$\dot{x} := M(x)\frac{\partial H}{\partial x} = (\Phi^{-1})\frac{\partial}{\partial x}(f^t P(x) f). \tag{26}$$

Next, we consider the relationship between the realization of (26) and the stability problem.

We say a matrix M is (strictly) dissipative if

$$M + M^T \leq 0, \quad (< 0).$$

LEMMA 3.6. *1. If a matrix M is dissipative and invertible then its inverse is also dissipative;*

2. If M is a strictly dissipative matrix, then M is invertible. Moreover, M^{-1} is also strict dissipative.

Proof. 1. Since M is dissipative

$$x^T M x \leq 0, \quad \forall x \in \mathbb{R}^n$$

Set $y = M^{-1}x$, then

$$y^T M^{-T} y = y^T M^{-1} y \leq 0, \quad \forall y \in \mathbb{R}^n.$$

So M^{-1} is also dissipative.

2. Decompose $M = K - R$, with K skew-symmetric and R symmetric and positive definite. Assume M is not invertible. Then there exists $x \in \mathbb{R}^n \backslash \{0\}$ such that

$$x^T M = 0.$$

It follows that

$$x^T M x = x^T (K - R) x = -x^T R x = 0,$$

which is a contradiction. So M is invertible.

Next assume $M^{-1} = \tilde{K} - \tilde{R}$ is not strict dissipative, i.e., there exists an $x \neq 0$ such that

$$0 = -x^T \tilde{R} x = x^T M^{-1} x.$$

Let $y = M^{-1} x \neq 0$. Then

$$x^T M^{-1} x = y^T M^T M^{-1} M y = y^T M^T y = y^T M y = -y^T R y = 0,$$

which is a contradiction again. So M^{-1} is strict dissipative. □

Consider the stability problem via the pseudo-Hamiltonian realization in Lemma 3.6.

THEOREM 3.7. *Assume H1 holds for system (4). 1. If there exists a positive definite matrix, $P(x) > 0$, such that*

$$\Phi(x) := P(x) J_f(x) + J_f^T(x) P(x) + f^T(x) \ltimes \left(\frac{\partial P(x)}{\partial x}\right)$$

is a locally invertible and dissipative matrix, then the origin is a stable equilibrium point.

If $\Phi(x)$ is globally invertible and dissipative, and the function, $f^T(x)P(x)f(x)$, is radially unbounded, then the system is globally stable at the origin.

2. If $\Phi(x)$ is locally strict dissipative, the origin is an asymptotically stable equilibrium point.

If $\Phi(x)$ is globally strict dissipative, and the function, $f^T(x)P(x)f(x)$, is radially unbounded, then the system is globally asymptotically stable at the origin.

Proof. Using Lemma 3.5, for both case 1 and case 2 we have a pseudo-Hamiltonian realization of the system (4) as

$$\dot{x} = M(x) \frac{\partial H}{\partial x}(x), \tag{27}$$

where $H(x) = f^T(x)P(x)f(x)$ and $M(x) = \Phi^{-1}(x)$. According to H1, $f(x) = 0$ implies $x = 0$ (locally or globally respectively to 1 or 2). Hence $H(x)$ is a Lyapunov function. Now since $\Phi(x)$ is dissipative (strict dissipative respectively), by Lemma 3.6 so is $M(x)$. Then

$$\dot{H}(x) = dH M(x) \frac{\partial H}{\partial x} \leq 0,$$

the local (global) stability is assured.

Next, when $M(x)$ is strict dissipative, we have to show that $\dot{H}(x) < 0$, for $x \neq 0$. Since $M(x)$ is strict dissipative, $\dot{H}(x) = 0$ implies $dH(x) = 0$. Since $\frac{\partial H}{\partial x}(x) = \Phi(x) f(x)$, and $\Phi(x)$ is non-singular, $dH(x) = 0$ implies $f(x) = 0$, and the later implies $x = 0$. □

REMARK 3.8. *In the above Theorem, when $P(x) = P > 0$, $\Phi(x)$ becomes $J_f^T P + P J_f$. When there exists $c > 0$ such that $\Phi(x) < -cI$, it is easy to show that $f^T(x)Pf(x)$, is radially unbounded. So the origin is globally asymptotically stable. This is the Krasovskii's theorem [12]. So the above is a generalization of Krasovskii's theorem. To see that the above Theorem is much more general we assume a system is globally asymptotically stable and the corresponding Lyapunov function, $L(x)$, is C^2. Moreover, assume the mapping $\pi : x \mapsto f(x)$ is a diffeomorphism. Then from the proof of Proposition 3.4, one sees that $L(x)$ can be expressed as $L(x) = f^T(x)P(x)f(x)$. That is Theorem 3.7 is applicable to such kind of systems, while the Krasovskii's Theorem doesn't work.*

Casimir function method is an useful tool in the stability and stabilization analysis of Hamiltonian systems [16], [7]. Using Casimir functions we may choose different Hamiltonian functions to represent same system. It gives us more freedom to choose a suitable Hamiltonian function as a Lyapunov function, or a storage function, etc.

Given a structure matrix $M(x)$, we define a *pseudo-Poisson bracket* as

$$\{F(x), G(x)\} = dFM(x)\frac{\partial G(x)}{\partial x}, \quad \forall F(x), G(x) \in C^\infty(\mathbb{R}^n). \tag{28}$$

Then for a given $G(x) \in C^\infty(\mathbb{R}^n)$ we can define a vector field, called the *pseudo-Hamiltonian vector field* generated by $G(x)$ and denoted by X_G, as

$$L_{X_G} F(x) = \{F(x), G(x)\}, \quad \forall F(x) \in C^\infty(\mathbb{R}^n). \tag{29}$$

A function $C(x)$ is called a *left (right) Casimir function* if

$$\{C(x), F(x)\} = 0, \quad (\{F(x), C(x)\} = 0) \quad \forall F(x) \in C^\infty(\mathbb{R}^n).$$

If a left Casimir function is also a right Casimir function, it is called a Casimir function.

When $M(x)$ is either symmetric or skew-symmetric matrix, a left (or right) Casimir function is also a Casimir function.

A right Casimir function, $C_r(x)$, may be added to the Hamiltonian function H, which will not affect the structure of the system. But for the new Hamiltonian function $\tilde{H} = H + C_r$, the derivative of the Lyapunov function along the trajectory is changed to $\frac{d}{dt}(\tilde{H}) = \frac{d}{dt}(H) + \{C_r, H\}$. Choosing suitable C_r may turn the Hamiltonian function to meet our requirement.

A left Casimir function, $C_l(x)$, may be added to the Hamiltonian function H, to form a new storage function as $S(x) = H(x) + C_l(x)$. It has the same derivative as H along the trajectory.

EXAMPLE 3.9. Consider the system (4) we denote $f = col(f^1, f^2)$. Note that we always use supscripts for blocks, and subscripts for individual components.

Assume

$$J_{11} = \frac{\partial f^1}{\partial x^1}, \quad f^1(x) \in \mathbb{R}^k, \ \ x^1 \in \mathbb{R}^k$$

is nonsingular. A local coordinate change can be obtained as

$$z_i = \begin{cases} f_i, & i = 1, \cdots, k \\ x_i, & i = k+1, \cdots, n. \end{cases}$$

Then

1. Locally we have

$$f^2(x) = \Psi(x) f^1(x) + \phi(x^2) x^2. \tag{30}$$

To prove (30), note that if $h(x)$ is a smooth function with $h(0) = 0$, then

$$h(x) = \int_0^1 \left. \frac{\partial h(r, x_2, \cdots, x_n)}{\partial r} \right|_{r = x_1 t} dt x_1 + R_2(x_2, \cdots, x_n) := a_1(x) x_1 + R_2.$$

Applying this to R_2 etc. inductively, we have

$$h(x) = a_1(x) x_1 + a_2(x_2, \cdots, x_n) x_2 + \cdots + a_n(x_n) x_n,$$

which implies (30).

2. There is a local pseudo-Hamiltonian realization with the Hamiltonian function $H(x) = \frac{1}{2}((f^1)^T f^1 + (x^2)^T x^2)$, such that

$$\dot{x} = \begin{pmatrix} J_{11}^{-T} & 0 \\ \Psi(x) J_{11}^{-T} - \phi(x^2) J_{12}^T J_{11}^{-T} & \phi(x^2) \end{pmatrix} \frac{\partial H}{\partial x}. \tag{31}$$

3. If $x \to (f^1, x^2)$ is a global coordinate transformation, then the realization is global.

4. For system (31) $C(x)$ is a right Casimir function iff $C(x) = C(x^2)$ and $\phi(x^2) \frac{\partial C(x)}{\partial x} = 0$. $C(x)$ is a left Casimir function iff $dC(x) \in D^{\perp}$, where

$$D = Span\ col \begin{pmatrix} I & 0 \\ \Psi(f_1) & \phi(x^2) \end{pmatrix}.$$

5. If (31) is (strict) dissipative then the system (4) is (asymptotically) stable. If (31) is a global realization and $\|f^1(x)\| \to \infty$ as $\|x\| \to \infty$, the system (4) is globally (asymptotically) stable. □

4. Stabilization via Feedback Dissipative Realization. We consider a pseudo-Hamiltonian system as

$$\begin{cases} \dot{x} = M(x) \frac{\partial H}{\partial x} + g(x) u = (K(x) - R(x) + T(x)) \frac{\partial H(x)}{\partial x} + g(x) u \\ y = g^T \frac{\partial H(x)}{\partial x}, \end{cases} \tag{32}$$

where $K(x)$ is skew symmetric and $R(x)$ and $T(x)$ are symmetric and positive semi-definite, and all $M(x)$, $K(x)$, $R(x)$, and $T(x)$ are C^r on $\mathbb{R} \backslash \{0\}$, $H(x)$, $g(x)$ are C^r on $\mathbb{R}^n$.

DEFINITION 4.1. *System (32) is said to have a feedback (strict) dissipative realization if there exists a state feedback*

$$u = \alpha(x) + v$$

such that the closed-loop system becomes a dissipative pseudo-Hamiltonian system as

$$\begin{cases} \dot{x} = (\tilde{K}(x) - \tilde{R}(x))\frac{\partial H}{\partial x} + gv \\ y = g^T \frac{\partial H}{\partial x}, \end{cases} \tag{33}$$

where $\tilde{K}$ is skew symmetric and $\tilde{R} \geq 0$ is positive semi-definite ($\tilde{R} > 0$ is positive definite).

We call (33) a dissipative pseudo-Hamiltonian system because it is easy to verify that as $H(x) > 0$ the system (33) is a dissipative system with $H(x)$ as its the storage function and the passivity supply rate $s(u, y) = u^T y$ [25]. Conversely, we want to show that (33) covers a large class of the dissipative systems.

PROPOSITION 4.2. *Assume the system (5) is a dissipative system with a C^r ($r \geq 2$) storage function, $S(x)$ with $S'(0) = 0$. Moreover, assume the Hessian matrix, $Hess(S(x))$ is nonsingular as $x \neq 0$. Then it can be expressed as a dissipative pseudo-Hamiltonian system.*

Proof. Using Taylor expansion we have that

$$\frac{\partial S(x)}{\partial x} = S'(0) + Hess(S(\xi))x = Hess(S(\xi))x, \quad \text{where } \xi_k \in (0, x_k), \quad k = 1, \cdots, n.$$

Hence $\frac{\partial S(x)}{\partial x} \neq 0$, as $x \neq 0$. According to Proposition 2.2, system (5) can be expressed as

$$\dot{x} = M(x)\frac{\partial S(x)}{\partial x} + gu$$

Since ([25] p 38) $S_x(x)f(x) \leq 0$, we have

$$dS(x)M(x)\frac{\partial S(x)}{\partial x} = x^T Hess^T(S(\xi))M(x)Hess(S(\xi))x \leq 0, \quad \forall x \in \mathbb{R}^n.$$

It follows that $M(x) \leq 0$. □

To use the stabilization method proposed in [15] [6], we have to convert the controlled pseudo-Hamiltonian systems into the dissipative type systems. We consider the following state feedback control

$$u = E(x)\frac{\partial H}{\partial x} + v.$$

As an immediate consequence of the definition, we have

PROPOSITION 4.3. *System (32) has a dissipative type realization (around an equilibrium point x_0), iff there exists an $m \times n$ matrix $E(x)$, such that the following matrix is negative semi- definite(locally).*

$$g(x)E(x) + E^T(x)g^T(x) + (M(x) + M^T(x)) \leq 0. \tag{34}$$

We are particularly interested in the case when both $M(x)$ and $g = (g_1 \ \cdots \ g_m)$ are constant. In this case we seek a particular output feedback control of the form

$$u = E\frac{\partial H}{\partial x} + v,$$

where E is a constant matrix.

Let $P = \frac{1}{2}(M + M^T)$. Then we have the following corollary.

COROLLARY 4.4. *System (32) with constant M and g has a dissipative type realization if there exists an $m \times n$ matrix E such that the following matrix is negative semi-definite :*

$$gE + E^T g^T + 2P \leq 0. \tag{35}$$

Note that we can decompose $P = -R + T$, with positive semi-definite R and T. When both R and T have minimum rank, the decomposition is unique. Assuming $span\{col(T)\} \subset span\{col(g)\}$, it is easy to find E which satisfies (35). In fact, if $T = g\alpha$ we can simply choose $E = -\alpha$.

Now we assume $\tilde{K}(x) = K(x)$, which provides a skew-symmetric structure matrix as required in many cases [25]. That is the skew-symmetric part is not changeable. Then we have

THEOREM 4.5. *Assume $g_i(x)$, $i = 1, \cdots, k$ are linearly independent, then system (32) has a feedback dissipative realization with $\tilde{K}(x) = K(x)$, iff there exists a continuous function $\lambda(x)$ such that*

$$\lambda(x)g(x)(g(x))^T - T(x) \geq 0. \tag{36}$$

Proof. (Sufficiency) We choose the control as

$$u = -\lambda(x)g^T\frac{\partial H}{\partial x} + v. \tag{37}$$

Then the feedback system becomes

$$\dot{x} = (K(x) - R(x) - (\lambda(x)g(x)g^T(x) - T(x)))\frac{\partial H}{\partial x} + gv.$$

Necessity: Since $g\alpha$ has the form as $g\alpha = E(x)\frac{\partial H}{\partial x}$, left-multiply both sides by $(g^Tg)^{-1}g^T$ we have

$$\alpha = (g^Tg)^{-1}g^TE(x)\frac{\partial H}{\partial x} := \xi\frac{\partial H}{\partial x}.$$

To keep the skew-symmetric part K unchanged, we need

$$(g\xi)^T = \xi^Tg^T = g\xi.$$

Express ξ as

$$\xi^T = g\xi g(g^T g)^{-1} := g\phi.$$

Now we have

$$g\phi^T g^T = g\xi = (g\xi)^T = g\phi g^T.$$

That is

$$g(\phi g^T - \phi^T g^T) = 0.$$

Using the fact that $g_i,\ i = 1, \cdots, m$ are linearly independent, we have $\phi g^T = \phi^T g^T$. Using similar argument again, we have $\phi = \phi^T$. Now by the definition, no cancellation will happen between R and T. So the feedback system is dissipative iff

$$g\phi g^T + T \leq 0.$$

Now let $\lambda(x) = -\|\phi(x)\|$. Then we have $\lambda(x)I_m - \phi(x) \leq 0$. It follows that $g(\lambda I_m - \phi)g^T \leq 0$. Hence

$$g(\lambda I_m - \phi)g^T + g\phi g^T + T = \lambda g g^T + T \leq 0,$$

which complete the proof. □

Above result is convenient in use. But it is restricted because the Hamiltonian function is known and fixed. In the following a Hamiltonian function can be constructed.

THEOREM 4.6. *The system (5) has a (strict) dissipative realization if J_f is invertible and there exist two positive definite constant matrices P and R such that*

$$J_f^{-T} P + P J_f^{-1} - P g R g^T P \leq 0 \quad (< 0). \tag{38}$$

Proof. Take $H = \frac{1}{2} f^T P f$ as a Hamiltonian function. Since J_f is invertible we have a pseudo-Hamiltonian realization as

$$\dot{x} = P^{-1} J_f^{-T} \frac{\partial H}{\partial x} + gu. \tag{39}$$

Choosing control as

$$u = -R g^T \frac{\partial H}{\partial x} + v, \tag{40}$$

the closed-loop system becomes

$$\dot{x} = (P^{-1} J_f^{-T} - g R g^T) \frac{\partial H}{\partial x} + gv. \tag{41}$$

Now to get a (strict) dissipative realization it is required that

$$P^{-1}J_f^{-T} + J_f^{-1}P^{-1} - gRg^T \leq 0 \quad (< 0), \tag{42}$$

which is equivalent to (38). $\square$

REMARK 4.7. *1. In linear case it is degenerated to the Algebraic Riccati equation. If (A, B) is controllable and A is invertible, then (A^{-1}, B) is also controllable. Hence, it is well know that the positive definite solution P always exists.*

2. The invertibility of A is not important because as long as (A, B) is controllable, we may first use pre-control E such that $A + BE$ is invertible.

3. In non-linear case, instead of the invertibility, we may assume $(J_f|_0, g(0))$ is controllable. In this case we can find a constant matrix K such that for the pre-feedback systems the Jacobian matrix of $f + gE$ is at least locally non-singular around the origin.

4. In linear case to solve (38) we can choose $Q < 0$ ($Q \leq 0$) and solve the algebraic Riccati equation

$$A^{-T}P + PA^{-1} - PBRB^TP = Q.$$

The following results are well known:

4.1. If (A, B) is controllable, then for $Q < 0$ there is a unique positive definite solution $P > 0$;

4.2. If (A, B) is controllable $R = C^TC$ and (A, C) is observable, then for $Q \leq 0$ there exists a unique solution $P > 0$.

4.3. If (A, B) is stabilizable and (A, C) is detectable then there exists a unique solution $P \geq 0$. Moreover, in all three cases $Re(eig(A - BB^TP)) < 0$.

In practical application, a question arises as: How to find a solution of (38)? It seems not so easy. One way to solve this problem is as the following: First we may give any constant $Q > 0$ to solve the constant algebraic Riccati equation:

$$J_f^{-T}(0)P + PJ_f^{-1}(0) - Pg(0)Rg(0)^TP = Q. \tag{43}$$

Let $A = J_f(0)$, and $B = g(0)$. If (A, B) is completely controllable, then for any $Q > 0$ we can find an unique solution of $P > 0$. By continuity, at least locally (38) is satisfied. Then we can check whether it is globally satisfied.

It is interesting to solve (38) for some particular cases. For instance, we have

PROPOSITION 4.8. *Assume for a given range $D \subset \mathbb{R}^n$, $g = B$ is a constant matrix, and*

$$J_f^{-1} = A + G\Sigma(x)F, \quad \Sigma^T(x)\Sigma(x) \leq I, \quad \forall x \in D,$$

where $A = J_f^{-1}(0)$ and G, F are constant matrices, and $\Sigma(x)$ is a matrix function. Then, a positive definite matrix P, satisfying (38) for all $x \in D$ is obtained by solving *the algebraic Riccati equation*

$$A^TP + PA + P(GG^T - BRB^T)P + F^TF = -Q. \tag{44}$$

Proof. Using the particular form of J_f^{-1}, (38) becomes

$$\begin{aligned} &J_f^{-T}P + PJ_f^{-1} - PBRB^TP = A^TP + PA - PBRB^T + F^T\Sigma^TG^TP + PG\Sigma F \\ &\le A^TP + PA - PBRB^T + (PG\Sigma - F^T)(PG\Sigma - F^T)^T + PGG^TP + F^TF \le -Q. \end{aligned}$$

□

Next, we consider the problem of strict realization of the system (32). That is, to find a feedback control $u = \alpha(x) + v$, such that the system (32) becomes (33) with positive definite $\tilde{R}(x) > 0$. We have the following

PROPOSITION 4.9. *1. A sufficient condition for system (32) to have a dissipative realization is*

$$Span\ col\{T\} \subset Span\{g\}. \tag{45}$$

2. A necessary condition for (32) to have a strict dissipative realization is

$$Span\ col\{R\} + Span\{g\} = \mathbb{R}^n. \tag{46}$$

3. (45) and (46) form a sufficient condition for system (32) to have a strict dissipative realization.

Proof. 1. (45) implies that there exists a smooth matrix $E(x)$ with proper dimension such that $T = gE$. Then $u = -E\frac{\partial H}{\partial x}$ makes the closed-loop system to be dissipative.

2. Assume $dim(span\ col\{R\} + span\{g\}) < n$. Then there exists a vector $X \in \mathbb{R}^n\backslash\{0\}$ such that $X \in ker(R) \cap ker(g)$. Now assume $gu = \xi\frac{\partial H}{\partial x}$, then $\xi \in Span\{g\}$. Say, $\xi = gN$, then

$$X^T(K - R + T + gN)X = X^TTX \ge 0,$$

which is a contradiction.

3. Choose $u = -(g^T + E)\frac{\partial H}{\partial x}$, where E is as in the proof of 1. We have only to show the negativeness of $K - R + T - (gg^T + gE)$ point-wise. For a given point we may express R as

$$R = \begin{pmatrix} R_0 & 0 \\ 0 & 0 \end{pmatrix},$$

where R_0 is non-singular. Correspondingly, we split g as $g = (g_1,\ \ g_2)$, where $dim(g_1) = dim(R_0)$. Now we have

$$gg^T = \begin{pmatrix} g_1g_1^T & g_1g_2^T \\ g_2g_1^T & g_2g_2^T \end{pmatrix}.$$

Condition (46) implies that g_2 has full row rank. For any $X \in \mathbb{R}^n \backslash \{0\}$, we split $X = (X_1\ X_2)^T$ with $dim(X_1) = dim(R_0)$. Then we have

$$X^T(K - R + T - (gg^T + gE))X = -X^T RX - X^T gg^T X = X_1^T R_0 X_1 - X^T gg^T X.$$

Set it to be zero, then $X_1 = 0$ and it follows that

$$X_2^T g_2 g_2^T X_2 = 0.$$

But $g_2 g_2^T$ is non-singular. Hence $X_2 = 0$. □

REMARK 4.10. *For a strict dissipative realization (46) is not sufficient and (45) is not necessary. Let*

$$R = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

If we set $g = (1\ 1)^T$, it is easy to verify that no matter how to choose E, $-R+T+gE$ can not be negative definite. Hence, (46) is not sufficient. If we set $g = (1\ 2)^T$, (45) is not satisfied. But if we choose $E = (0\ -2)$, then $-R+T+gE$ represents a negative definite quadratic form.

Finally, we consider a dissipative realization of a general control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i. \tag{47}$$

When $m = 1$, the problem was discussed in [23]. A generalization is the following

PROPOSITION 4.11. *Assume the system*

$$\dot{x} = \sum_{i=1}^{m} g_i(x)u_i \tag{48}$$

has a smooth state feedback $u_i = \xi_i(x)$ such that the closed-loop system (48) is (globally) asymptotically stable, then (47) has a (global) feedback dissipative pseudo-Hamiltonian realization with controls smooth on ($\mathbb{R}^n \backslash \{0\}$) $U \backslash \{0\}$, where U is a neighborhood of the origin. Consequently, (47) is (globally) stabilizable.

Proof. By Lyapunov inverse theorem, we can assume there exist $\xi_i(x)$ and a Lyapunov function $V(x) > 0$ such that $L_{\tilde{g}} V < 0$, where $\tilde{g} = \sum_{i=1}^{m} g_i(x)\xi_i$. Then we can construct a control as $u = w + v$, where

$$w = \begin{cases} -\frac{1}{L_g V(x)}(\xi f^T - \xi\xi^T g^T)\frac{\partial V(x)}{\partial x}, & x \neq 0 \\ 0, \quad x = 0. \end{cases} \tag{49}$$

Then a straightforward computation shows that the closed-loop system can be expressed as a dissipative pseudo-Hamiltonian system

$$\dot{x} = M(x)\frac{\partial V(x)}{\partial x} + gv = (K(x) - R(x))\frac{\partial V(x)}{\partial x} + gv, \tag{50}$$

where the skew-symmetric $K(x)$ and the positive semi-definite $R(x)$ are

$$K(x) = \begin{cases} \left[\frac{f(x)\tilde{g}^T - \tilde{g}f^T}{L_{\tilde{g}}V(x)}\right], & x \neq 0 \\ 0, & x = 0; \end{cases} \qquad R(x) = \begin{cases} \left[\frac{\tilde{g}\tilde{g}^T}{L_{\tilde{g}}V(x)}\right], & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Moreover, it is ready to verify that the derivative of $V(x)$ along the trajectory of the system (50) with $v = 0$ is

$$\dot{V} = L_{\tilde{g}}V(x) < 0.$$

So (47) is asymptotically stabilized by the control $u = w$. □

REMARK 4.12. *1. If $L_gV(0) = 0$, the control will be unbounded at zero. So the control can be used for practical stabilization (trajectory enters any given neighborhood of the origin). Similar to [23], we may theoretically assume that $L_gV(x) \neq 0$ to avoid the unbounded control.*

2. The stabilization claim is related to Sontag's approach [24].

5. L_2 Disturbance Attenuation. In this section we consider a pseudo-Hamiltonian system as the following:

$$\begin{cases} \dot{x} = M(x)\frac{\partial H}{\partial x} + g_1(x)u + g_2(x)w \\ z = C(x), \quad x \in \mathbb{R}^n, \ \ u \in R^m, w \in R^q, z \in R^p, \end{cases} \tag{51}$$

where z is the penalty signal and w is the disturbance.

The L_2 disturbance attenuation problem of pseudo-Hamiltonian system has been discussed in [22]. The L_2 disturbance attenuation problem can be described as follows: Given a penalty signal $z = q(x)$, a disturbance attenuation level $\gamma > 0$ and a desired equilibrium $x_0 \in \mathbb{R}^n$, find a feedback control law $u = k(x)$, a positive storage function $V(x)$ and a non-negative definite function $Q(x)$ such that the γ-dissipation inequality

$$\dot{V} + Q(x) \leq \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\}, \quad \forall w \tag{52}$$

holds along all trajectories of the closed-loop system (51) with a designed feedback law.

REMARK 5.1. *1. The property (52) ensures the following performance [3].*

P1. The L_2 gain from w to z is less than the preassigned γ;

P2. If $Q(x)$ is positive definite, i.e., $Q(x) \neq 0$, $\forall x \neq x_0$, the closed-loop system with vanished w is asymptotically stable at x_0;

P3. If w is square integrable, then x is uniformly bounded. If there is a κ_∞ function κ such that

$$Q(x) + \|z\|^2 \geq \kappa(\|x - x_0\|), \quad \forall x,$$

then a bounded $w(t)$ gives bounded states $x(t)$.

2. Note that if the system is output detectable with penalty output z, i.e., $z = 0$ implies $x \to 0$ (as $t \to \infty$), then the conclusion of P2 remains true without the positive definiteness assumption of $Q(x)$. In this section we will always assume either one holds.

First, we decompose g_2 as

$$g_2 = g_2^1 + g_2^2, \quad \text{where} \quad g_2^1 \in Span(g_1), \quad g_2^2 \in (Span(g_1))^{\perp}.$$

Then we can express g_2 as

$$g_2 = g_1\xi + \tilde{g}_2, \quad \text{where} \quad g_2^1 = g_1\xi, \quad g_2^2 = \tilde{g}_2.$$

If $\tilde{g}_2 = 0$ and $\xi = I_m$, the problem is reduced to the case discussed in [22], where the dissipative realization is assumed. The following is our main result for H_∞ control design for dissipative Hamiltonian systems.

THEOREM 5.2. *For system (51) assume*
A1. $M(x) = K(x) - R(x)$ with $R(x) \geq 0$ and $H(x) > 0$;
A2. $z = h(x)g_1^T \frac{\partial H}{\partial x}$.
A3. There is a $\eta > 0$ such that $-R(x) + \eta\tilde{g}_2(\tilde{g}_2)^T \leq 0$.
Then the L_2 disturbance attenuation objective for $\gamma \geq \frac{1}{\sqrt{2\eta}}$ is achieved by the following feedback control law:

$$u = -[\frac{1}{2}(h^T h)g_1^T + \frac{1}{2\gamma^2}(\xi\xi^T g_1^T + 2\xi(\tilde{g}_2)^T)]\frac{\partial H}{\partial x}. \tag{53}$$

Proof. Using control (53), a straightforward computation shows that

$$\begin{aligned} \frac{dH}{dt} &= -dHR\frac{\partial H}{\partial x} + dHg_1u + dHg_2w \\ &+\frac{1}{2}dHg_1h^T h(g_1)^T\frac{\partial H}{\partial x} + dHg_1u + \frac{1}{2\gamma^2}dHg_2(g_2)^T\frac{\partial H}{\partial x}. \end{aligned} \tag{54}$$

Note that

$$\begin{aligned} dHg_2g_2^T\frac{\partial H}{\partial x} &= dH(g_1\xi + \tilde{g}_2)(\xi^t g_1^T + (\tilde{g}_2)^T)\frac{\partial H}{\partial x} \\ &= dH[g_1\xi\xi^T g_1^T + g_1\xi(\tilde{g}_2)^T + \tilde{g}_2\xi^T g_1^T + \tilde{g}_2(\tilde{g}_2)^T]\frac{\partial H}{\partial x} \\ &= dHg_1(\xi\xi^T g_1^T + 2g_1\xi(\tilde{g}_2)^T)\frac{\partial H}{\partial x} + dH\tilde{g}_2(\tilde{g}_2)^T\frac{\partial H}{\partial x}. \end{aligned} \tag{55}$$

Plugging (55) into (54) and using control (53) yield

$$\dot{H} = -dH(R - \frac{1}{2\gamma^2}\tilde{g}_2(\tilde{g}_2)^T)\frac{\partial H}{\partial x} - \frac{1}{2}\|\gamma w - \frac{1}{\gamma}(g_2)^T\frac{\partial H}{\partial x}\|^2 + \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2). \tag{56}$$

As $\gamma \geq \frac{1}{\sqrt{2\eta}}$, we can set

$$Q(x) = dH(R - \frac{1}{2\gamma^2}\tilde{g}_2(\tilde{g}_2)^T))\frac{\partial H}{\partial x} \geq 0.$$

Then we have

$$\dot{H} + Q(x) \leq \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2), \tag{57}$$

which completes the proof. □

Theorem 5.2 assures only that the closed-loop system with vanished w is stable at x_0. If either $Q(x)$ is positive definite or system is output detectable, the closed-loop system with vanished w is asymptotically stable. In fact, we have another sufficient condition.

PROPOSITION 5.3. *In Theorem 5.2, if x_0 is the only equilibrium point of the free system(i.e., $u = 0$ and $w = 0$), and A3 is replaced by*

$$-R(x) + \eta\tilde{g}_2(\tilde{g}_2)^T - \frac{1}{2}ghh^Tg^T < 0. \tag{58}$$

Then the conclusion of Theorem 5.2 holds. Moreover, the closed-loop system with vanished w is asymptotically stable.

Proof. From (56) we have

$$\begin{aligned}\dot{H} &\leq dH(-R(x) + \eta\tilde{g}_2(\tilde{g}_2)^T)\tfrac{\partial H}{\partial x} - \tfrac{1}{2}\|z\|^2\\ &= dH(-R(x) + \eta\tilde{g}_2(\tilde{g}_2)^T - ghh^Tg^T)\tfrac{\partial H}{\partial x}.\end{aligned} \tag{59}$$

Using (58), $\dot{H} = 0$ implies $dH = 0$. But x_0 is the only equilibrium point of the free system. So $\frac{\partial H}{\partial x} \neq 0$ $(x \neq x_0)$. That is, $\dot{H}$ is negative definite. The conclusion follows. □

Note that if (58) holds at x_0 the conclusion is local, and if (58) holds globally, the conclusion is global.

In Theorem 5.2, assumptions A1 and A2 are natural. We would like to analyze A3 a little bit more.

LEMMA 5.4. *A3 implies that*

$$Span\{\tilde{g}_2\} \subset Span\ col\{R\}.$$

Proof. Without loss of generality we assume

$$R(x) = \begin{pmatrix} R_0(x) & 0 \\ 0 & 0 \end{pmatrix}$$

where $R_0(x)$ is non-singular. Correspondingly, we express

$$\tilde{g}_2 = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}.$$

Then

$$\tilde{g}_2(\tilde{g}_2)^T = \begin{pmatrix} G_1G_1^T & G_1G_2^T \\ G_2G_1^T & G_2G_2^T \end{pmatrix}.$$

Now it is obvious that if $G_2 \neq 0$, A3 can never be true. □

Under a coordinate change, we may assume

$$\tilde{g}_2 = \begin{pmatrix} G_0(x) \\ 0 \end{pmatrix},$$

where G_0 is non-singular. Then we have

PROPOSITION 5.5. *Let the restriction of $R(x)$ on the subspace spanned by $\tilde{g}_2$ be $R_g(x)$. Then*

(60)
$$\eta = \min_x \min_\lambda \{\lambda(x) \in \ eig(G_0^{-1} R_g G_0^{-T})\}.$$

Proof. Since

$$R_g(x) - \eta G_0(x) G_0^T(x) \geq 0$$

which is equivalent to

$$G_0^{-1}(x) R_g(x) G_0^{-T}(x) - \eta I \geq 0.$$

The conclusion follows. □

6. Dissipative realization and H_∞ Control of Excitation System. As an application, we consider the excitation control systems. The model for excitation control of the single-machine infinite bus power system with silicon-controlled rectifier direct excitor is described as [22]

(61)
$$\begin{cases} \dot{\delta} = \omega - \omega_0 \\ \dot{\omega} = \frac{\omega_0}{M} P_m - \frac{D}{M}(\omega - \omega_0) - \frac{\omega_0 E_q' V_s}{M x_{d\Sigma}'} sin\delta \\ \dot{E}_q' = -\frac{1}{T_d'} E_q' + \frac{1}{T_{d0}} \frac{x_d - x_d'}{x_{d\Sigma}'} V_s \cos\delta + \frac{1}{T_{d0}} V_f \end{cases}$$

where δ: the rotor angle; ω: rotor speed; E_q': internal transient voltage; P_M: mechanical power; M: inertia coefficient of a generator; D: damping constant; $P_e = \frac{E_q' V_s}{x_{d\Sigma}'} sin\delta$: active electrical power; T_d': stator closed loop time constant; T_{d0}: excitation circuit time constant; x_d: stator circuit self-inductor resister; x_d': stator circuit transient resister; V_f: voltage of the field circuit of a generator.

Consider $u = V_f$ as the control, and set $x_1 = \delta$, $\quad x_2 = \omega - \omega_0$, $\quad x_3 = E_q'$ and denote $a = \frac{\omega_0}{M} P_m$, $b = \frac{D}{M}$, $c = \frac{\omega_0 V_s}{M x_{d\Sigma}'}$, $d = \frac{1}{T_d'}$, $e = \frac{1}{T_{d0}} \frac{x_d - x_d'}{x_{d\Sigma}'} V_s$, and $h = \frac{1}{T_{d0}}$, then system (61) becomes

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = f + gu = \begin{pmatrix} x_2 \\ a - bx_2 - cx_3 \sin x_1 \\ -dx_3 + e \cos x_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} u. \tag{62}$$

Now we consider all the possible Hamiltonian realizations. Since g is constant, we can ignore it. For f in (62) equation (16) becomes

(63)

$$\begin{pmatrix} 1 & -b & 0 & 0 & cx_3 \cos x_1 & e \sin x_1 & 0 & 0 & 0 \\ 0 & -c \sin x_1 & -d & 0 & 0 & 0 & 0 & cx_3 \cos x_1 & e \sin x_1 \\ 0 & 0 & 0 & 0 & -c \sin x_1 & -d & -1 & b & 0 \end{pmatrix} X = 0.$$

We are looking for a constant solution, but the terms with different degrees of x_1 and x_3 in (63) provide an infinite number of linear equations. If we consider the constant terms and terms linear in x_1 and x_3 only, we get the following linear system:

$$\begin{pmatrix} 1 & -b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -d & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -d & -1 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & e & 0 & 0 & 0 \\ 0 & -c & 0 & 0 & 0 & 0 & 0 & 0 & e \\ 0 & 0 & 0 & 0 & -c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} X = 0. \tag{64}$$

It is easy to check that the other terms in x do not provide any new equations. Let $X = (n_{11}, n_{12}, \cdots, n_{33})$. Since the rank of the coefficient matrix of (64) is 7, we may simply choose $n_{33} = \frac{c}{e}; n_{21} = \alpha$. Then, up to a constant coefficient, the solution of (64) is given by

$$M^{-1} = N = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} = \begin{pmatrix} b & 1 & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & \frac{c}{e} \end{pmatrix} \quad M = \mu \begin{pmatrix} 0 & 1 & 0 \\ \alpha & -b & 0 \\ 0 & 0 & \frac{e\alpha}{c} \end{pmatrix}$$

where μ is an arbitrary coefficient. We may simply set $\mu = 1$. Now for any non-zero α M provides a Hamiltonian realization. If we consider the dissipative realization, it is easy to verify that the only possible solution is $\alpha = -1$.

Using this M, system (62) can be rewritten as

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = \left(\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c} \end{pmatrix} \right) \begin{pmatrix} -a + cx_3 \sin x_1 \\ x_2 \\ -c \cos x_1 + \frac{cd}{e} x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} u. \tag{65}$$

Choosing a suitable output, the dissipative realization of (65) is expressed as

$$\begin{cases} \begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = (K(x) - R(x))\frac{\partial H}{\partial x} + g(x)u \\ y = g^T \frac{\partial H}{\partial x} \end{cases} \tag{66}$$

where

$$K(x) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad R(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c} \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}$$

and then

$$y = -\frac{c}{T_{d0}} \cos x_1 + \frac{cd}{T_{d0} e} x_3,$$

and

$$H(x) = -cx_3 \cos x_1 - ax_1 + \frac{cd}{2e} x_3^2 + \frac{1}{2} x_2^2. \tag{67}$$

This is exactly the form presented in [5]. Based on the above argument, we know that the dissipative realization of the excitation control system (65) without feedback is unique (up to a constant coefficient).

Next, we consider the feedback dissipative realization. Recalling Corollary 4.4, one sees easily that a trivial solution for equation (35) is $K = (0\ 0\ k_3)$, where $k_3 \leq 0$. Now $gK = diag(0\ 0\ g_3 k_3)$, which means when the feedback is allowed, new Hamiltonian function, H_e, can be chosen as $H_e = H + (ax_3 + b)$. Based on this consideration, we may use the following Hamiltonian function [22], which differs from (67) in a linear function of x_3.

$$H_e(x) = \frac{1}{2} x_2^2 + b_L x_3 (\cos x_{1e} - \cos x_1) - P(x_1 - x_{1e}) + \frac{b_L c_T}{c_L} (x_3 - x_{3e})^2 \tag{68}$$

and a pre-control is used as

$$\bar{u}(t) = c_T x_{3e} - c_L \cos x_{1e}. \tag{69}$$

It was shown in [27] that

$$x_{1e} = \frac{1}{2} \sin^{-1}(2ad/ec); \quad x_{3e} = (e/d) cos(x_1),$$

and locally $H > 0$ and reaches its minimum at the point $(x_{1e}, 0, x_{3e})$. Equivalently, under control (69), locally $H_e > 0$ and reaches its minimum at $(x_{1e}, 0, x_{3e})$.

Finally, we consider the problem of the L_2 disturbance attenuation of the excitation control system. Based on the above argument and the disturbance system as in [21], we can formulate the system as

$$\begin{cases} \dot{x} = (K - R) \frac{\partial H}{\partial x}_e + g_1 u + g_2 w \\ z = h(x) g_1^t \frac{\partial H}{\partial x}_e \end{cases} \tag{70}$$

where K, R and g_1 are as in equation (66). $w = (w_1, w_2)$ are the disturbances, and

$$g_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Using the data in [28] as $D = 3.0$; $M = 7.6$; $\omega_0 = 50\pi$; $V_s = 20$; $x'_{d\Sigma} = 0.36$; $Pm = 100$; $T'_d = 5.0$; $T_{d0} = 5.0$; $x_d = 0.9$; $x'_d = 0.36$; $k = 1$; $l = 1$; $x_{10} = 1.7$; $x_{20} = 5$; $x_{30} = 7$, we have

$$R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3/7.6 & 0 \\ 0 & 0 & \frac{0.2\times 7.6\times 0.36}{50\pi\times 20} \end{pmatrix}, \quad g_1 = \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix},$$

$$g_2^1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad g_2^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \tilde{g}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix},$$

and $\xi = (1\ 0)$. To use Theorem 5.2, we verify the assumptions. A1 and A2 are obviously true. To make assumption A3 true, i.e.,

$$-R + \eta \tilde{g}_2 (\tilde{g}_2)^T \le 0.$$

The largest η is obtained as $\eta = 3/7.6$. Hence

$$\gamma \ge \frac{1}{\sqrt{2\eta}} = 1.125.$$

We conclude that for this excitation control system the disturbance attenuation problem is solvable only for the L_2 gain $\gamma \ge 1.125$. Moreover, using (53) a feasible control is

$$u = \bar{u} - [\frac{1}{2}(h^T h)(0\ 0\ 0.2) + \frac{1}{2\gamma^2}(0\ 2\ 1)]\frac{\partial H_e}{\partial x}.$$

Note that this system is output detectable [27]. So x_0 is asymptotically stable.

We omit the computer simulating results here due to the space limitation. But the performance is very encouraging.

7. Conclusion. The problem of pseudo-Hamiltonian realization was considered. First of all, several sufficient conditions were provided for a dynamic system to be convertible to a pseudo-Hamiltonian system. Some related stability results were obtained via revealing the relationship between Hamiltonian functions and Lyapunov functions. Among them, a generalization of the Krasovskii's Theorem is obtained. It is also proved that an affine nonlinear system is stabilizable if the system, obtained by setting its drift term to be zero, is stabilizable. Then the realization with constant structure matrix and the feedback dissipative realization of a control system were investigated. The results obtained were applied to the stabilization problem. Next, some conditions were obtained for a pseudo-Hamiltonian realization with constant structure matrix. Meanwhile, it was shown that under certain conditions the following systems are mutually convertible: stable (asymptotically stable) system with dissipative (strict dissipative) pseudo-Hamiltonian system; dissipative affine control system with dissipative pseudo-Hamiltonian system.

Finally, the problem of disturbance attenuation of the pseudo-Hamiltonian systems was studied. It was shown that a class of pseudo-Hamiltonian systems the disturbance attenuation problem is solvable and an estimation of the boundary of the L_2 gain is obtained. As an application example the results were implemented to the excitation control of power systems.

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