For Tyrone Duncan

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Tyrone Duncan has made seminal contributions in the field of Filtering, Stochastic Control and the interface of probability theory and geometry. In his pioneering thesis, he presented a fundamental theory of nonlinear filtering and was instrumental in introducing the modern theory of martingales as developed by Doob, Meyer, Kunita-Watanabe and others in the study of nonlinear filtering. As he said to one of us, he was fortunate to learn the subject from Watanabe in the course on Martingale theory which Watanabe gave at Stanford when Tyrone was a graduate student. The celebrated stochastic partial differential equation describing the evolution of the unnormalized conditional density of the state given the observations was introduced in his thesis (independently by Mortensen and Zakai). As an outgrowth of the ideas presented in his thesis, he was the first to give a causal representation for the Radon-Nikodym derivative of the probability measure representing the signal plus noise with respect-to the probability measure representing noise only-the celebrated likelihood ratio formula obtained via the Girsanov Theorem. In important work, he also gave the formula for mutual information between a stochastic signal (the state) and an observation where the state is observed in additive white noise.

One of us is a direct beneficiary of Tyrone's research in this field. Tyrone's other research which belongs in the same period is his joint work with Pravin Varaiya on existence theorems for Optimal Stochastic Control, one of the first general theorems on existence questions. Motivated by problems in Quantum Field Theory, Irving Segal, in the sixties, undertook a fundamental investigation of the Absolute Continuity of Gaussian measures obtained as a translation of white noise measure in a canonical Hilbert space setting. These ideas were further developed by Leonard Gross in the framework of Abstract Wiener spaces. Tyrone was one of the first to appreciate the importance of this work to Stochastic Systems Theory. One of us distinctly remembers his lecture on this subject given at Imperial College in the mid-seventies.

The study of Stochastic Differential Equations in manifolds was initiated by K. Itô and continued by R. Gangolli, in his MIT thesis, and by H.P. McKean. Amongst system theorists, Tyrone was again the first to recognize the importance of this work and made important initial contributions to the theory of stochastic integrals on Riemannian Manifolds and to the study of Estimation and Stochastic Control on Riemannian Manifolds. He presented explicit solutions to estimation and stochastic control problems defined on manifolds, some of the few explicit solutions known for Estimation and Stochastic Control. Brownian motion and the heat equation play important roles in Geometry. The deep and sustained work of Bismut on Geometry, especially his celebrated proof of the Atiyah-Singer Index Theorem using Probabilistic Methods, has explicitly highlighted this connection. What is less well known is that Tyrone's proof of the Index Theorem predates the work of Bismut. This is a striking example of the originality of Tyrone's thinking. His work on the representation of Affine Lie algebras using Brownian motion is another example of his unique capability of making connections between seemingly different intellectual fields.

Tyrone spent the years 1979 and 80 at Harvard. One of us remembers with nostalgia the Friday afternoon excursions to the Ha'Penny Pub (which alas does not exist any more) with Chris Byrnes, Peter Caines and Tyrone where some of the most exciting intellectual discussions on Systems Theory took place. This was also a time of great ferment and excitement in the field. One of the topics of discussion was the role of Algebraic Geometry and Topology in understanding the structure of systems. Here, for the first time, one of us learnt about the Bott Periodicity Theorem and what it had to say about the invariants of linear systems. Tyrone's own research at this time is reflected in his joint work with Chris Byrnes on topological invariants of systems. As mentioned previously, the contributions of Tyrone Duncan to stochastic control are fundamental and bear some characteristics which are unique in the literature. He has the ability to work in a vast domain and explore it in all its aspects. He progresses from simple to more complex situations, develops the mathematical theory, considers the numerical aspects, and looks at very concrete applications. This may appear to be the natural way to conduct research in Applied Mathematics, but it is not so common, and certainly not carried out as systematically as in the works of Tyrone Duncan. This is best illustrated by considering the field of Adaptive Control, on which he decided to work twenty years ago, and in which he is now a well established leader. With Bozenna Pasik-Duncan and some other colleagues and students he has covered all aspects of this domain. Adaptive Control is the problem of making decisions when the system has unknown parameters. The basic challenge is to obtain a design which eventually converges to the exact values of the parameters and the corresponding optimal control. Tyrone's articles are among the main references in the field. Following the systematic approach described above, he has considered all possible situations and all possible problems of interest: proven consistency, obtained rates of convergence both for finite dimensional systems, infinite dimensional systems, differential delay systems, fully and partially observable systems, linear and nonlinear systems, boundary control, discrete time and continuous time, numerical approximations and computational aspects. From the very beginning he has looked at models motivated by applications: portfolio and consumption models, manufacturing models, risk-sensitive models.

More recently (in 2000) Tyrone started working on stochastic models with fractional Brownian motion as inputs. One knows that the traditional Brownian motion models have serious limitations in modeling extreme phenomenon which occur in the crash of financial markets or in communication networks. This motivates the use of fractional Brownian motion in modeling such phenomena. Unfortunately, these processes lack the nice properties of Wiener processes. They are not Markovian, they are not semi-martingales. That creates a serious hurdle which has, over a long period, prevented progress in their use for applications.

With his energy and renowned talent Tyrone is systematically covering this new domain in all aspects, with remarkable success. He has developed basic techniques like stochastic calculus and Itô's formula and then studied large classes of stochastic differential equations with fractional Brownian motion inputs. This is probably the most comprehensive set of results concerning stochastic dynamical systems driven by Fractional Brownian motion. At the same time he has considered many engineering problems, whose solutions are well known for the case of stochastic differential equations with Brownian motion inputs but totally open for fractional Brownian motion inputs. Tyrone has used these models in problems of identification, estimation and control with remarkable success. In addition, he has studied many applied models (Finance, ATM traffic, queues) where modeling with fractional Brownian motion inputs is particularly relevant. This work is still in its infancy and will undoubtedly undergo significant development in Tyrone's capable hands.

This short account of Tyrone's scientific work demonstrates the uniqueness and versatility of his contributions. As a person, he is known for his gentleness and kindness. He is a gentleman and a scholar, a faithful friend and an esteemed colleague. With Bozenna and their charming daughter Dominique, they form a model family, as well as a great team, and will continue to make great contributions to Science and Engineering.